Nonlinear Fiber Optics and its Applications in Optical Signal Processing

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Outline

- Introduction
- Self-Phase Modulation
- Cross-Phase Modulation
- Four-Wave Mixing
- Stimulated Raman Scattering
- Stimulated Brillouin Scattering
- Concluding Remarks
Introduction

Fiber nonlinearities

- Studied during the 1970s.
- Ignored during the 1980s.
- Feared during the 1990s.
- Are being used in this decade.

Objective:

- Review of Nonlinear Effects in Optical Fibers.
Major Nonlinear Effects

• Self-Phase Modulation (SPM)
• Cross-Phase Modulation (XPM)
• Four-Wave Mixing (FWM)
• Stimulated Raman Scattering (SRS)
• Stimulated Brillouin Scattering (SBS)

Origin of Nonlinear Effects in Optical Fibers

• Ultrafast third-order susceptibility $\chi^{(3)}$.
• Real part leads to SPM, XPM, and FWM.
• Imaginary part leads to SBS and SRS.
Self-Phase Modulation

- Refractive index depends on optical intensity as (Kerr effect)
  \[ n(\omega, I) = n_0(\omega) + n_2 I(t). \]

- Frequency dependence leads to dispersion and pulse broadening.

- Intensity dependence leads to nonlinear phase shift
  \[ \phi_{NL}(t) = \frac{2\pi}{\lambda} n_2 I(t)L. \]

- An optical field modifies its own phase (thus, SPM).

- Phase shift varies with time for pulses (chirping).

- As a pulse propagates along the fiber, its spectrum changes because of SPM.
Nonlinear Phase Shift

• Pulse propagation governed by Nonlinear Schrödinger Equation

\[ i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0. \]

• Dispersive effects within the fiber included through \( \beta_2 \).
• Nonlinear effects included through \( \gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}} \).
• If we ignore dispersive effects, solution can be written as

\[ A(L,t) = A(0,t) \exp(i\phi_{\text{NL}}), \quad \text{where} \quad \phi_{\text{NL}}(t) = \gamma L |A(0,t)|^2. \]

• Nonlinear phase shift depends on input pulse shape.
• Maximum Phase shift: \( \phi_{\text{max}} = \gamma P_0 L = L/L_{\text{NL}} \).
• Nonlinear length: \( L_{\text{NL}} = (\gamma P_0)^{-1} \sim 1 \text{ km for } P_0 \sim 1 \text{ W.} \)
SPM-Induced Chirp

- Super-Gaussian pulses: \( P(t) = P_0 \exp\left(-\left(t/T\right)^{2m}\right) \).
- Gaussian pulses correspond to the choice \( m = 1 \).
- Chirp is related to the phase derivative \( d\phi/dt \).
- SPM creates new frequencies and leads to spectral broadening.
SPM-Induced Spectral Broadening

- First observed in 1978 by Stolen and Lin.
- 90-ps pulses transmitted through a 100-m-long fiber.
- Spectra are labelled using $\phi_{\text{max}} = \gamma P_0 L$.
- Number $M$ of spectral peaks: $\phi_{\text{max}} = (M - \frac{1}{2}) \pi$.

- Output spectrum depends on shape and chirp of input pulses.
- Even spectral compression can occur for suitably chirped pulses.
SPM-Induced Spectral Narrowing

- Chirped Gaussian pulses with $A(0,t) = A_0 \exp\left[-\frac{1}{2}(1+iC)(t/T_0)^2\right]$.
- If $C < 0$ initially, SPM produces spectral narrowing.
SPM: Good or Bad?

- SPM-induced spectral broadening can degrade performance of a lightwave system.

- Modulation instability often enhances system noise.

On the positive side . . .

- Modulation instability can be used to produce ultrashort pulses at high repetition rates.

- SPM often used for fast optical switching (NOLM or MZI).

- Formation of standard and dispersion-managed optical solitons.

- Useful for all-optical regeneration of WDM channels.

- Other applications (pulse compression, chirped-pulse amplification, passive mode-locking, etc.)
Modulation Instability

Nonlinear Schrödinger Equation

\[ i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0. \]

- CW solution unstable for anomalous dispersion ($\beta_2 < 0$).
- Useful for producing ultrashort pulse trains at tunable repetition rates [Tai et al., PRL 56, 135 (1986); APL 49, 236 (1986)].
Modulation Instability

- A CW beam can be converted into a pulse train.
- Two CW beams at slightly different wavelengths can initiate modulation instability and allow tuning of pulse repetition rate.
- Repetition rate is governed by their wavelength difference.
- Repetition rates $\sim 100$ GHz realized by 1993 using DFB lasers (Chernikov et al., APL 63, 293, 1993).
Optical Solitons

- Combination of SPM and anomalous GVD produces solitons.
- Solitons preserve their shape in spite of the dispersive and nonlinear effects occurring inside fibers.
- Useful for optical communications systems.

- Dispersive and nonlinear effects balanced when $L_{NL} = L_D$.
- Nonlinear length $L_{NL} = 1/(\gamma P_0)$; Dispersion length $L_D = T_0^2/|\beta_2|$.
- Two lengths become equal if peak power and width of a pulse satisfy $P_0 T_0^2 = |\beta_2|/\gamma$. 
Fundamental and Higher-Order Solitons

- NLS equation: \( i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0 \).

- Solution depends on a single parameter: \( N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|} \).

- Fundamental \((N = 1)\) solitons preserve shape:
  \[
  A(z, t) = \sqrt{P_0} \text{sech}(t/T_0) \exp(iz/2L_D).
  \]

- Higher-order solitons evolve in a periodic fashion.

![Graphs showing power vs. time and distance for different values of N](attachment:image.png)
Stability of Optical Solitons

- Solitons are remarkably stable.
- Fundamental solitons can be excited with any pulse shape.

Gaussian pulse with $N = 1$.
Pulse eventually acquires a ‘sech’ shape.

- Can be interpreted as temporal modes of a SPM-induced waveguide.
- $\Delta n = n_2 I(t)$ larger near the pulse center.
- Some pulse energy is lost through dispersive waves.
Cross-Phase Modulation

- Consider two optical fields propagating simultaneously.
- Nonlinear refractive index seen by one wave depends on the intensity of the other wave as
  \[ \Delta n_{NL} = n_2(|A_1|^2 + b|A_2|^2). \]
- Total nonlinear phase shift:
  \[ \phi_{NL} = \left(\frac{2\pi L}{\lambda}\right)n_2[I_1(t) + bI_2(t)]. \]
- An optical beam modifies not only its own phase but also of other copropagating beams (XPM).
- XPM induces nonlinear coupling among overlapping optical pulses.
XPM-Induced Chirp

- Fiber dispersion affects the XPM considerably.
- Pulses belonging to different WDM channels travel at different speeds.
- XPM occurs only when pulses overlap.
- Asymmetric XPM-induced chirp and spectral broadening.

![Graph showing intensity over frequency](image)
XPM: Good or Bad?

- XPM leads to interchannel crosstalk in WDM systems.
- It can produce amplitude and timing jitter.

On the other hand …

XPM can be used beneficially for

- Nonlinear Pulse Compression
- Passive mode locking
- Ultrafast optical switching
- Demultiplexing of OTDM channels
- Wavelength conversion of WDM channels
XPM-Induced Crosstalk

- A CW probe propagated with 10-Gb/s pump channel.
- Probe phase modulated through XPM.
- Dispersion converts phase modulation into amplitude modulation.
- Probe power after 130 (middle) and 320 km (top) exhibits large fluctuations (Hui et al., JLT, 1999).
XPM-Induced Pulse Compression

- An intense pump pulse is copropagated with the low-energy pulse requiring compression.
- Pump produces XPM-induced chirp on the weak pulse.
- Fiber dispersion compresses the pulse.
XPM-Induced Mode Locking

• Different nonlinear phase shifts for the two polarization components: nonlinear polarization rotation.

\[ \phi_x - \phi_y = (2\pi L / \lambda)n_2[(I_x + bI_y) - (I_y + bI_x)]. \]

• Pulse center and wings develop different polarizations.

• Polarizing isolator clips the wings and shortens the pulse.

• Can produce \(~100\) fs pulses.
Synchronous Mode Locking

- Laser cavity contains the XPM fiber (few km long).
- Pump pulses produce XPM-induced chirp periodically.
- Pulse repetition rate set to a multiple of cavity mode spacing.
- Situation equivalent to the FM mode-locking technique.
Four-Wave Mixing (FWM)

- FWM is a nonlinear process that transfers energy from pumps to signal and idler waves.
- FWM requires conservation of (notation: $E = \text{Re}[Ae^{i(\beta z - \omega t)}]$)
  - **Energy** $\omega_1 + \omega_2 = \omega_3 + \omega_4$
  - **Momentum** $\beta_1 + \beta_2 = \beta_3 + \beta_4$
- Degenerate FWM: Single pump ($\omega_1 = \omega_2$).
Theory of Four-Wave Mixing

- Third-order polarization: $\mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)}: \mathbf{E} \mathbf{E} \mathbf{E}$ (Kerr nonlinearity).

$$\mathbf{E} = \frac{1}{2} \hat{x} \sum_{j=1}^{4} F_j(x,y) A_j(z,t) \exp[i(\beta_j z - \omega_j t)] + \text{c.c.}$$

- The four slowly varying amplitudes satisfy

$$\frac{dA_1}{dz} = \frac{in_2 \omega_1}{c} \left[ \left( f_{11} |A_1|^2 + 2 \sum_{k \neq 1} f_{1k} |A_k|^2 \right) A_1 + 2 f_{1234} A_2^* A_3 A_4 e^{i\Delta k z} \right]$$

$$\frac{dA_2}{dz} = \frac{in_2 \omega_2}{c} \left[ \left( f_{22} |A_2|^2 + 2 \sum_{k \neq 2} f_{2k} |A_k|^2 \right) A_2 + 2 f_{2134} A_1^* A_3 A_4 e^{i\Delta k z} \right]$$

$$\frac{dA_3}{dz} = \frac{in_2 \omega_3}{c} \left[ \left( f_{33} |A_3|^2 + 2 \sum_{k \neq 3} f_{3k} |A_k|^2 \right) A_3 + 2 f_{3412} A_1 A_2 A_4^* e^{-i\Delta k z} \right]$$

$$\frac{dA_4}{dz} = \frac{in_2 \omega_4}{c} \left[ \left( f_{44} |A_4|^2 + 2 \sum_{k \neq 4} f_{4k} |A_k|^2 \right) A_4 + 2 f_{4312} A_1 A_2 A_3^* e^{-i\Delta k z} \right]$$
Simplified Scalar Theory

- Linear phase mismatch: $\Delta k = \beta_3 + \beta_4 - \beta_1 - \beta_2$.

- Overlap integrals $f_{ijkl} \approx f_{ij} \approx 1/A_{\text{eff}}$ in single-mode fibers.

- Full problem quite complicated (4 coupled nonlinear equations).

- Undepleted-pump approximation $\Rightarrow$ two linear coupled equations:

  - Using $A_j = B_j \exp[2i\gamma(P_1 + P_2)z]$, the signal and idler satisfy:
    \[
    \frac{dB_3}{dz} = 2i\gamma\sqrt{P_1 P_2} B_4^* e^{-i\kappa z}, \quad \frac{dB_4}{dz} = 2i\gamma\sqrt{P_1 P_2} B_3^* e^{-i\kappa z}.
    \]

- Total phase mismatch: $\kappa = \beta_3 + \beta_4 - \beta_1 - \beta_2 + \gamma(P_1 + P_2)$.

- Nonlinear parameter: $\gamma = n_2 \omega_0 / (cA_{\text{eff}}) \sim 10 \text{ W}^{-1}/\text{km}$.

- Signal power $P_3$ and Idler power $P_4$ are much smaller than pump powers $P_1$ and $P_2$ ($P_n = |A_n|^2 = |B_n|^2$).
General Solution

- Signal and idler fields satisfy:

\[
\frac{dB_3}{dz} = 2i\gamma \sqrt{P_1 P_2} B_4^* e^{-i\kappa z}, \quad \frac{dB_4^*}{dz} = -2i\gamma \sqrt{P_1 P_2} B_3 e^{i\kappa z}.
\]

- General solution when both the signal and idler are present at \( z = 0 \):

\[
B_3(z) = \{ B_3(0) [\cosh(gz) + (i\kappa/2g) \sinh(gz)] \\
+ (i\gamma/g) \sqrt{P_1 P_2} B_4^*(0) \sinh(gz) \} e^{-i\kappa z/2}
\]

\[
B_4^*(z) = \{ B_4^*(0) [\cosh(gz) - (i\kappa/2g) \sinh(gz)] \\
- (i\gamma/g) \sqrt{P_1 P_2} B_3(0) \sinh(gz) \} e^{i\kappa z/2}
\]

- If an idler is not launched at \( z = 0 \) (phase-insensitive amplification):

\[
B_3(z) = B_3(0) [\cosh(gz) + (i\kappa/2g) \sinh(gz)] e^{-i\kappa z/2}
\]

\[
B_4^*(z) = B_3(0) (-i\gamma/g) \sqrt{P_1 P_2} \sinh(gz) e^{i\kappa z/2}
\]
Gain Spectrum

- Signal amplification factor for a FOPA:
  \[ G(\omega) = \frac{P_3(L,\omega)}{P_3(0,\omega)} = \left[ 1 + \left( 1 + \frac{\kappa^2(\omega)}{4g^2(\omega)} \right) \sinh^2[g(\omega)L] \right]. \]

- Parametric gain: 
  \[ g(\omega) = \sqrt{4\gamma^2P_1P_2 - \kappa^2(\omega)/4}. \]

- Wavelength conversion efficiency:
  \[ \eta_c(\omega) = \frac{P_4(L,\omega)}{P_3(0,\omega)} = \left( 1 + \frac{\kappa^2(\omega)}{4g^2(\omega)} \right) \sinh^2[g(\omega)L]. \]

- Best performance for perfect phase matching (\( \kappa = 0 \)):
  \[ G(\omega) = \cosh^2[g(\omega)L], \quad \eta_c(\omega) = \sinh^2[g(\omega)L]. \]
Parametric Gain and Phase Matching

In the case of a single pump:
\[ g(\omega) = \sqrt{(\gamma P_0)^2 - \kappa^2(\omega)/4}. \]

Phase mismatch \( \kappa = \Delta k + 2\gamma P_0 \)

Parametric gain maximum when \( \Delta k = -2\gamma P_0 \).

- Linear mismatch: \( \Delta k = \beta_2 \Omega^2 + \beta_4 \Omega^4/12 + \cdots \), where \( \Omega = \omega_s - \omega_p \).
- Phase matching realized by detuning pump wavelength from fiber’s ZDWL slightly such that \( \beta_2 < 0 \).
- In this case \( \Omega = \omega_s - \omega_p = (2\gamma P_0/|\beta_2|)^{1/2} \).
Highly Nondegenerate FWM

- Some fibers can be designed such that $\beta_4 < 0$.
- If $\beta_4 < 0$, phase matching is possible for $\beta_2 > 0$.
- $\Omega$ can be very large in this case.

![Graph showing frequency shift versus $|\beta_4|$](image)
FWM: Good or Bad?

- FWM leads to interchannel crosstalk in WDM systems.
- It generates additional noise and degrades system performance.

On the other hand ...

FWM can be used beneficially for

- Optical amplification and wavelength conversion
- Phase conjugation and dispersion compensation
- Ultrafast optical switching and signal processing
- Generation of correlated photon pairs
Parametric Amplification

- FWM can be used to amplify a weak signal.
- Pump power is transferred to signal through FWM.
- Peak gain $G_p = \frac{1}{4} \exp(2\gamma P_0 L)$ can exceed 20 dB for $P_0 \sim 0.5$ W and $L \sim 1$ km.
- Parametric amplifiers can provide gain at any wavelength using suitable pumps.
- Two pumps can be used to obtain 30–40 dB gain over a large bandwidth (>40 nm).
- Such amplifiers are also useful for ultrafast signal processing.
- They can be used for all-optical regeneration of bit streams.
Single- and Dual-Pump FOPAs

- Pump close to fiber’s ZDWL
- Wide but nonuniform gain spectrum with a dip

- Pumps at opposite ends
- Much more uniform gain
- Lower pump powers (∼0.5 W)
Optical Phase Conjugation

- FWM generates an idler wave during parametric amplification.
- Its phase is complex conjugate of the signal field \( A_4 \propto A_3^* \) because of spectral inversion.
- Phase conjugation can be used for dispersion compensation by placing a parametric amplifier midway.
- It can also reduce timing jitter in lightwave systems.
Stimulated Raman Scattering

- Scattering of light from vibrating silica molecules.
- Amorphous nature of silica turns vibrational state into a band.
- Raman gain spectrum extends over 40 THz or so.

- Raman gain is maximum near 13 THz.
- Scattered light red-shifted by 100 nm in the 1.5 \( \mu \text{m} \) region.
Raman Threshold

- Raman threshold is defined as the input pump power at which Stokes power becomes equal to the pump power at the fiber output:

\[ P_s(L) = P_p(L) \equiv P_0 \exp(-\alpha_p L). \]

- \( P_0 = I_0 A_{\text{eff}} \) is the input pump power.

- For \( \alpha_s \approx \alpha_p \), threshold condition becomes

\[ P_{s0}^{\text{eff}} \exp(g_R P_0 L_{\text{eff}}/A_{\text{eff}}) = P_0, \]

- Assuming a Lorentzian shape for the Raman-gain spectrum, Raman threshold is reached when (Smith, Appl. Opt. 11, 2489, 1972)

\[ \frac{g_R P_{th} L_{\text{eff}}}{A_{\text{eff}}} \approx 16 \quad \implies \quad P_{th} \approx \frac{16 A_{\text{eff}}}{g_R L_{\text{eff}}}. \]
Estimates of Raman Threshold

Telecommunication Fibers

- For long fibers, \( L_{\text{eff}} = \left[ 1 - \exp(-\alpha L) \right]/\alpha \approx 1/\alpha \approx 20 \text{ km} \)
  for \( \alpha = 0.2 \text{ dB/km} \) at 1.55 \( \mu \text{m} \).

- For telecom fibers, \( A_{\text{eff}} = 50-75 \ \mu \text{m}^2 \).

- Threshold power \( P_{\text{th}} \sim 1 \text{ W} \) is too large to be of concern.

- Interchannel crosstalk in WDM systems because of Raman gain.

Yb-doped Fiber Lasers and Amplifiers

- Because of gain, \( L_{\text{eff}} = \left[ \exp(gL) - 1 \right]/g > L \).

- For fibers with a large core, \( A_{\text{eff}} \sim 1000 \ \mu \text{m}^2 \).

- \( P_{\text{th}} \) exceeds 10 kW for short fibers (\( L < 10 \text{ m} \)).

- SRS may limit fiber lasers and amplifiers if \( L \gg 10 \text{ m} \).
SRS: Good or Bad?

- Raman gain introduces interchannel crosstalk in WDM systems.
- Crosstalk can be reduced by lowering channel powers but it limits the number of channels.

On the other hand …

- Raman amplifiers are a boon for WDM systems.
- Can be used in the entire 1300–1650 nm range.
- EDFA bandwidth limited to \(~\)40 nm near 1550 nm.
- Distributed nature of Raman amplification lowers noise.
- Needed for opening new transmission bands in telecom systems.
Stimulated Brillouin Scattering

- Scattering of light from acoustic waves.
- Becomes a stimulated process when input power exceeds a threshold level.
- Low threshold power for long fibers ($\sim 5 \text{ mW}$).
- Most of the power reflected backward after SBS threshold is reached.
Brillouin Shift

- Pump produces density variations through electrostriction, resulting in an index grating which generates Stokes wave through Bragg diffraction.

- Energy and momentum conservation require:

\[ \Omega_B = \omega_p - \omega_s, \quad \vec{k}_A = \vec{k}_p - \vec{k}_s. \]

- Acoustic waves satisfy the dispersion relation:

\[ \Omega_B = v_A |\vec{k}_A| \approx 2v_A |\vec{k}_p| \sin(\theta/2). \]

- In a single-mode fiber \( \theta = 180^\circ \), resulting in

\[ \nu_B = \Omega_B/2\pi = 2n_p v_A / \lambda_p \approx 11 \text{ GHz}, \]

if we use \( v_A = 5.96 \text{ km/s}, \quad n_p = 1.45, \quad \text{and } \lambda_p = 1.55 \mu\text{m}. \)
Brillouin Gain Spectrum

- Measured spectra for (a) silica-core (b) depressed-cladding, and (c) dispersion-shifted fibers.
- Brillouin gain spectrum is quite narrow (∼50 MHz).
- Brillouin shift depends on GeO$_2$ doping within the core.
- Multiple peaks are due to the excitation of different acoustic modes.
- Each acoustic mode propagates at a different velocity $v_A$ and thus leads to a different Brillouin shift ($v_B = 2n_p v_A / \lambda_p$).
Brillouin Threshold

- Pump and Stokes evolve along the fiber as
  
  \[-\frac{dI_s}{dz} = g_B I_p I_s - \alpha I_s, \quad \frac{dI_p}{dz} = -g_B I_p I_s - \alpha I_p.\]

- Ignoring pump depletion, \(I_p(z) = I_0 \exp(-\alpha z).\)

- Solution of the Stokes equation:
  
  \[I_s(L) = I_s(0) \exp(g_B I_0 L_{\text{eff}} - \alpha L).\]

- Brillouin threshold is obtained from
  
  \[\frac{g_B P_{\text{th}} L_{\text{eff}}}{A_{\text{eff}}} \approx 21 \quad \Rightarrow \quad P_{\text{th}} \approx \frac{21 A_{\text{eff}}}{g_B L_{\text{eff}}}.\]

- Brillouin gain \(g_B \approx 5 \times 10^{-11} \text{ m/W}\) is nearly independent of the pump wavelength.
Estimates of Brillouin Threshold

Telecommunication Fibers

- For long fibers, $L_{\text{eff}} = \frac{1 - \exp(-\alpha L)}{\alpha} \approx \frac{1}{\alpha} \approx 20$ km for $\alpha = 0.2$ dB/km at 1.55 µm.

- For telecom fibers, $A_{\text{eff}} = 50$–75 µm².

- Threshold power $P_{th} \sim 1$ mW is relatively small.

Yb-doped Fiber Lasers and Amplifiers

- Because of gain, $L_{\text{eff}} = \frac{\exp(gL) - 1}{g} > L$.

- $P_{th}$ exceeds 20 W for a 1-m-long standard fibers.

- Further increase occurs for large-core fibers; $P_{th} \sim 400$ W when $A_{\text{eff}} \sim 1000$ µm².

- SBS is the dominant limiting factor at power levels $P_0 > 1$ kW.
Techniques for Controlling SBS

- Pump-Phase modulation: Sinusoidal modulation at several frequencies >0.1 GHz or with a pseudorandom bit pattern.
- Cross-phase modulation by launching a pseudorandom pulse train at a different wavelength.
- Temperature gradient along the fiber: Changes in $\nu_B = 2n_p\nu_A/\lambda_p$ through temperature dependence of $n_p$.
- Built-in strain along the fiber: Changes in $\nu_B$ through $n_p$.
- Nonuniform core radius and dopant density: mode index $n_p$ also depends on fiber design parameters ($a$ and $\Delta$).
- Control of overlap between the optical and acoustic modes.
- Use of Large-core fibers: A wider core reduces SBS threshold by enhancing $A_{\text{eff}}$. 
Concluding Remarks

- Optical fibers exhibit a variety of nonlinear effects.
- Fiber nonlinearities are feared by telecom system designers because they can affect system performance adversely.
- Fiber nonlinearities can be managed thorough proper system design.
- Nonlinear effects are useful for many device and system applications: optical switching, soliton formation, wavelength conversion, broadband amplification, channel demultiplexing, etc.
- New kinds of fibers have been developed for enhancing nonlinear effects (microstructured fibers with air holes).
- Nonlinear effects in such fibers are finding new applications.