Fig. 4 shows the variation of backscattered power due to the variation of dielectric thickness $h$ for normal incidence.

$$f = 9850 \text{GHz}, a = 0.5d, d = \lambda$$

The low backscatter for normal incidence can be explained as a consequence of eqn. 1 when $\theta_i = 0$, i.e. the diffraction angle will be 90°. The backscattered power is plotted by rotating the receiver along the circumference of a circle with the geometric centre of the plate as the centre in the horizontal plane (Fig. 5). The maximum diffracted power is obtained when the receiving angle becomes 90°. The same result is obtained when $\theta_r = 90^\circ$, i.e. when the incident ray is parallel to the plane of the grating.

Fig. 5 Scattered power against angle for normal incidence

$$f = 9840 \text{GHz}, a = 0.5d, h = 0.08\lambda$$

Conclusion: The problem of eliminating normal and near-normal incidence specular reflection is solved using a self-complementary strip grating. This could not be achieved using conventional rectangular metallic corrugations. Fabricating corrugations on the metal surface is a tedious process, which can be avoided by using strips on the dielectric sheet with a reflector. To reduce the multipath interference from buildings and to solve the problem of air traffic control systems at airports and instrument landing system interference etc., we may obtain better performance with the addition of easier construction technique and lower cost.

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EVALUATION OF CROSSTALK PENALTY IN MULTICHANNEL ASK HETERODYNE OPTICAL COMMUNICATION SYSTEMS

Indexing terms: Optical communications, Crosstalk

Introduction: One of the attractions of coherent communication systems is the possibility of simultaneous transmission of a large number of closely spaced channels using frequency-division multiplexing techniques. An important issue for such multichannel coherent systems concerns the minimum inter-channel spacing that must be maintained before the inter-channel crosstalk significantly degrades the system performance. Although this issue has recently been addressed both theoretically and experimentally,1-4 a detailed analysis of the effect of crosstalk on the system performance does not appear to have been carried out. In this letter we calculate the crosstalk-induced power penalty for two-channel ASK heterodyne systems employing envelope detection, and discuss the minimum channel spacing needed to keep the power penalty below a prespecified level. The extension to multichannel systems is straightforward, and the analysis can also be extended to the case of FSK systems. For multichannel ASK heterodyne systems, the theory predicts a minimum channel spacing of five times the bit rate if the design criterion is to keep the crosstalk penalty below 0.1 dB. The minimum channel spacing increases to five times the bit rate for multichannel systems when crosstalk from the nearest neighbours on both sides is included.

Theory: The output of a balanced heterodyne receiver can be written as

$$I = 2R \sqrt{P_{LO} P_1 m_1} \cos (2nf_{IF} t)$$
$$+ 2R \sqrt{P_{LO} P_2 m_2} \cos [2nf_{IF} (D + \phi) + N(t)]$$

(1)

where $R$ is the detector responsivity, $f_{IF}$ is the intermediate frequency, $D$ is the channel spacing, $P_{LO}$ is the local-oscillator power, $P_1$ is the signal-channel power and $P_2$ is the power of the interfering channel. The relative phase $\phi$ generally varies from bit to bit in a random manner. The ASK modulation is represented by $m_1$ and $m_2$, which take values of 1 and 0 depending on the bit pattern. $N(t)$ accounts for various noise sources (shot noise, circuit noise etc.) and is taken to be a Gaussian process with a white spectral density. The effect of laser phase noise (i.e. at the transmitter and the local oscillator) can be included by taking

$$f_{IF} = f_0 + \Delta f$$

(2)

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where $f_0$ is the average value and $\Delta f$ accounts for random fluctuations due to phase noise. In general $\Delta f$ changes randomly during a bit period.

The received signal is passed through a bandpass filter centred on $f_0$ and an envelope detector. Although the filter bandwidth $W$ is chosen judiciously to transmit most of the signal-channel energy while effectively blocking the neighbouring channels, the filter nevertheless intercepts a small fraction of the energy from the neighbouring channels. This constitutes the crosstalk signal that interferes with the post-detection processing of channel 1. The crosstalk depends on whether a '1' or a '0' is received by channel 2, and both possibilities should be considered. Since the two symbols are equally likely to occur, the bit error rate (BER) or the error probability $P_e$ is given by

$$P_e = \frac{1}{2} (P_{e1} + P_{e0})$$  (3)

where $P_{e1}$ and $P_{e0}$ are the error probabilities for the cases of receiving a '1' or a '0' in the interfering channel, respectively.

To evaluate $P_{e1}$, we note that the output of the envelope detector is given by

$$x = [(m_1 A + B \cos \phi + N_1^2) + (B \sin \phi + N_2)^2]^{1/2}$$  (4)

where $N_1$ and $N_2$ are the quadrature components of the noise, $A$ is the data signal and $B$ is the interfering signal due to crosstalk. $A$ and $B$ depend on the received power as well as on the bit rate and the filter bandwidth $W$. Their explicit expressions are

$$A = 2R \left[ (P_{L0} P_2)^{1/2} \int_{-W/2}^{W/2} \sin [\pi T_j (-\Delta f)] T_1 \, df \right]$$  (5)

and

$$B = 2R \left[ (P_{L0} P_2)^{1/2} \int_{-W/2}^{W/2} \sin [\pi T_j (D + \Delta f)] T_2 \, df \right]$$  (6)

where $B_j = 1/T_j$ is the bit rate of channel $j$, $\sin (x) = \sin (x)/x$ and we have assumed a rectangular input pulse and an ideal filter $[H(f) = 0 \text{ for } |f| > W/2]$. Both $A$ and $B$ are in general random variables because of frequency fluctuations $\Delta f$. In this letter we neglect the effect of laser phase noise by assuming $T_j \Delta f < 1$. The error probability $P_{e1}$ is given by

$$P_{e1} = \frac{1}{2} \int_{0}^{V_x} p_1(x) \, dx + \frac{1}{2} \int_{0}^{V_x} p_0(x) \, dx$$  (7)

where $p_1(x)$ and $p_0(x)$ are the probability densities for the '1' and '0' symbols in channel 1, respectively. More specifically, both are Rician distributions; i.e.

$$p_d(x) = \frac{x}{\sigma^2} f_0\left(\frac{Bx}{\sigma^2}\right) \exp\left(-\frac{x^2 + B^2}{2\sigma^2}\right)$$  (8)

and $p_1(x)$ is obtained by replacing $B$ by $(A^2 + B^2 + 2AB \cos \phi)^{1/2}$. Here $f_0(x)$ is the modified Bessel function of order zero and $\sigma^2$ is the noise variance (proportional to the filter bandwidth $W$).

The decision threshold $V_x$ in eqn. 7 is chosen to minimise $P_{e1}$. In the absence of crosstalk, $V_x = A/2$ for $A/\sigma > 1$, and we assume it to be the same even in the presence of crosstalk. This is generally also the case in practice. It can be shown that the contribution of the first term in eqn. 7 is smaller by about a factor of $A/\sigma$ compared to the second term, and can be neglected. This simplifies the calculation considerably since $P_e$ becomes independent of $\phi$. In the general case $P_e$ should be averaged over $\phi$. The second integral can be performed analytically with the result

$$P_{e1} = \frac{1}{4} \exp\left(-\frac{A^2}{8\sigma^2}\right) f(A, B, \sigma)$$  (9)

where

$$f(A, B, \sigma) = \exp\left(-\frac{B^2}{2\sigma^2}\right) \sum_{m=0}^{\infty} \frac{(2B/A)^m}{m!} 2^m$$  (10)

represents the enhancement of the error probability in the presence of crosstalk. $P_{e0}$ is obtained by setting $B = 0$ in eqn. 9, and is given by

$$P_{e0} \approx \frac{1}{4} \exp\left(-\frac{A^2}{8\sigma^2}\right)$$  (11)

This is the well known expression for the BER in the absence of crosstalk. Using eqns. 9 and 11 in eqn. 3, the BER of a two-channel ASK system is given by

$$P_e = \frac{1}{4} \exp\left(-\frac{A^2}{8\sigma^2}\right) [f(A, B, \sigma) + 1]$$  (12)

**Results:** Eqn. 12 was used to study the variation of $P_e$ with $A/\sigma$ for different values of the parameter $B/\sigma$. These BER curves show that a higher value of $A/\sigma$ is needed to maintain a certain BER in the presence of crosstalk, resulting in a power penalty. The penalty becomes significant when the crosstalk is comparable to the noise level. The power penalty can be calculated by noting that the BER for a single channel is given by

$$P_e = \frac{1}{4} \exp\left(-\frac{\bar{A}^2}{8\sigma^2}\right)$$  (13)

where $\bar{A}$ is the value of $A$ in the absence of crosstalk ($\bar{A}/\sigma = 12.65$ for a BER of $10^{-9}$). By setting $P_e = P_e\bar{A}/\bar{A}$ can be obtained. The power penalty (in decibels) is then given by

$$\Delta = 20 \log (A/\bar{A})$$  (14)

Fig. 1 shows the variation of $\Delta$ with the crosstalk at a given BER. The level of crosstalk (in decibels) is defined by

$$C = 10 \log (B/A)$$  (15)

The curve shown in Fig. 1 is universal (at a given BER) in the sense that the power penalty depends on a single crosstalk parameter $C$. Of course, the value of $C$ depends on the received power ratio $P_2/P_1$ as well as on the filter bandwidth $W$, channel spacing $D$ and the channel bit rates $B_1$ and $B_2$. Fig. 2 shows the variation of crosstalk $C$ with $D/B_2$ after assuming $P_1 = P_2$, $B_1 = B_2$ and $W = 2B_1$. The oscillatory structure is due to the sin function in eqns. 5 and 6. The crosstalk decreases with an increase in channel spacing and falls below $-20$ dB for $D/B_2 > 4$.

**Discussion:** From the standpoint of system design, the relevant question is how close two channels can operate without introducing a significant power penalty. If we assume that the maximum tolerable penalty $\Delta = 0.1$ dB, from Fig. 1 the crosstalk
talk should be < -17dB. Fig. 2 shows that C < -17dB if
D/B2 ≥ 3, i.e. the minimum channel spacing is about three
times the bit rate. In the case of multichannel operation the
dominant contribution to crosstalk comes from the nearest
neighbours located on each side of the signal channel. When
the analysis is extended to include both interfering channels,
the results show that a minimum channel spacing of five times
the bit rate6 can ensure a negligible crosstalk penalty. These
results assume equal received powers and equal bit rates for
the two channels. For P2 > P1, the crosstalk increases by a
factor of \(\sqrt{(P_2/P_1)}\), and a larger channel spacing would be
required. Similarly, the minimum channel spacing may
increase because of phase noise when the IF linewidth \(\Delta f_{IF}\)
becomes comparable to the bit rate. The results presented
here assume that \(\Delta f_{IF} < B_1\) and \(B_2\). The conclusions are
expected to remain valid for \(\Delta f_{IF} < 0.1B_1\), a condition often
satisfied in practice.

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BIREFRINGENT-FIBRE POLARISATION SPLITTERS

Indexing terms: Optical fibres, Optical couplers

A simple analytical study on fused tapered couplers com-
posed of birefringent fibres is presented. It is shown that the
polarisation splitting property of the coupler can remain
almost constant over a wide range of wavelengths when the
geometrical birefringence and the stress-induced birefrin-
gence in the coupler are properly balanced.

Introduction: Polarisation splitting has been observed in fused
tapered single-mode fibre couplers made of nonbirefringent fibres1-2 or birefringent fibres3-4. The operation of such
devices relies on the difference in the coupling coefficient between the two orthogonal polarisation modes.1-4 In a non-
birefringent-fibre coupler, this difference is caused purely by
the geometrical birefringence (or form birefringence) arising
from the noncircular cross-section of the coupler, and is found
to increase approximately with the square of the wave-
length.5-7 In a birefringent-fibre coupler, the stress-induced
birefringence can also cause polarisation splitting,4 and, as we
shall show, this effect diminishes with increasing wavelength.
It is thus expected that a balance of these two birefringences
in a coupler can make the polarisation splitting characteristic
of the coupler insensitive to wavelength variation.

Analysis and results: We consider a fused tapered birefringent-
fibre coupler with core radius \(p_1\) and cladding radius \(p_2\). The
refractive indices of the cladding and the surrounding material
are \(n_2\) and \(n_3\), respectively. For the sake of simplicity, we
assume that material birefringence is stress-induced only in the
core region. The refractive index of the cores is thus
\(n_{1y} = n_{1x}\), for the x-polarised mode and \(n_{1y} \neq n_{1x}\) for the y-
polarised mode, where we assume that \(n_{1y} > n_{1x}\). The polarisation
splitting property of the coupler is described by the difference
between the coupling coefficients of the x- and y-
polarised modes, \(C_x - C_y\), which can be written as

\[C_x - C_y = \delta C_G + \delta C_S\] (1)

where \(\delta C_G\) and \(\delta C_S\) represent the geometrical and stress-
induced effects, respectively. We assume that the effects of the
cores on \(\delta C_G\) are negligible, and thus have5-7

\[\delta C_G = q_0 \frac{(2\Delta \lambda)^{3/2}}{\rho_2 \nu^2} \] (2)

where \(V = 2\pi \rho_2 n_{3} (2\Delta \lambda)^{1/2}/\lambda\) is the normalised frequency with
\(\lambda\) the free-space wavelength, \(\Delta \lambda = (n_1^2 - n_2^2)/2n_2^2\) is the cladding-substrate index profile height, and \(q_0\) is a constant
depending on the cross-section of the coupler. The stress-
induced component \(\delta C_S\) can be easily derived from the results in Reference 6:

\[\delta C_S = q_s \frac{\mu_B V}{\rho_2^2 (2\Delta \lambda)^{1/2}}\] (3)

where \(B = n_{1y} - n_{1x}\) is the material birefringence and \(q_s\) is a structure-dependent constant. That \(\delta C_S\) increases linearly with
\(B\) at a given wavelength is consistent with the results in Ref-
ence 4. If we assume that \(q_0\) and \(q_s\) are both positive, it is
clear that \(\delta C_S\) is inversely proportional to \(V^2 (\lambda^2)\), while \(\delta C_G\) is propor-
tional to \(V (\lambda^2)\). However, we should point out that eqns. 2 and 3, which are accurate at small \(V\)-values,6 provide
only qualitative knowledge at large \(V\)-values. From eqns. 2
and 3 the \(V\)-value at which \(\delta C_G = \delta C_S\), denoted by \(V_x\), is
given by

\[V_x = \left[ \frac{q_0}{q_s} \left( \frac{\rho_2}{\rho_1} \right)^2 \frac{n_3 (2\Delta \lambda)^{1/2}}{B} \right]^{1/3}\] (4)

It follows that the minimum \(C_x - C_y\), where \(d(C_x - C_y)/dV = 0\), occurs at \(V = V_M\), where

\[V_M = 2^{1/3} V_x\] (5)

If the coupler is operated at \(V_M\), its polarisation splitting
property will be least sensitive to wavelength variation.

To quantify our discussion, we analyse a rectangular
coupler with square cores. This structure is a good model for
a practical stadium-shaped coupler.4-7 The constants \(q_0\) and
\(q_s\) are given by5,6

\[q_0 = \frac{3\pi^2 \rho_2^2}{(2\rho_2 + a)^3}\] (6)