Polarization changes of partially coherent pulses propagating in optical fibers

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1. INTRODUCTION
The degree of polarization reflects the vectorial nature of statistical electromagnetic fields, and it is an important measurable quantity that characterizes the evolution of partially coherent light fields [1–3]. The polarization of light may change on propagation as a result of various factors, including the nonlinearity and random birefringence of the medium [4–6]. In this paper, we focus on polarization changes of light induced by the source fluctuations. Although this topic attracted attention as early as 1973 [7], it was only after 1993 that it was fully developed in the context of partially coherent beam propagation in free space and in a linear nondispersive dielectric medium [8–10]. By now, coherence-induced polarization changes have been extensively studied for beams [11] and nonparaxial fields [12,13] in free space as well as for beams in turbulent atmosphere [14], and gradient-index fibers [15] within the framework of a recently formulated unified theory of coherence and polarization [3].

The vast majority of investigations of statistical optical fields to date have been concerned with stationary fields. At the same time, statistical optical pulses represent a wide class of partially coherent fields that find numerous applications in areas as diverse as optical imaging and fiber optics [6]. While a few studies on partially coherent pulses are available, they have, to our knowledge, been so far limited to the scalar case [16–23]. In this work, we consider propagation of partially coherent electromagnetic pulses in linear optical media. In particular, we focus on propagation-induced changes in the degree of polarization of partially coherent pulses launched inside single-mode optical fibers.

Propagation of fully coherent pulses along optical fibers has been studied extensively in the presence of dispersion as well as nonlinear effects [6]. Random birefringence of optical fibers has also attracted much attention in the past decade because the resulting polarization mode dispersion (PMD) has become a limiting effect in high-bit-rate and long-haul communication systems [5,6]. Such effects can be suppressed in the so-called polarization-maintaining fibers by introducing a relatively large constant birefringence. In this paper we consider propagation of partially coherent pulses along such fibers in the linear regime, i.e., we neglect the nonlinear effects but include both the group-velocity dispersion (GVD) and birefringence. The evolution of the degree of polarization of light in such fibers is determined by the interplay between the pulse spreading, which depends on the coherence properties of the two polarization components, and the spatial walk-off resulting from a birefringence-induced group-velocity mismatch. This circumstance makes the polarization dynamics quite different from the previously studied correlation-induced polarization changes of statistically stationary fields propagating in free space [8].

This work is organized as follows. In Section 2 we introduce the electromagnetic Gaussian Schell-model (GSM) and use it to obtain a general expression for the second-order correlation tensor at any point along the fiber. We then briefly discuss the state of polarization of the pulse in the source plane. In Section 3, we find an analytical expression for the degree of polarization across the temporal profile of the pulse at any point along the optical fiber. We then illustrate our general results with numerical examples and discuss their implications. The main conclusions are presented in Section 4.

2. SECOND-ORDER COHERENCY TENSOR
To describe partially coherent pulses, we choose a realization from the statistical ensemble of optical pulses and
employ the Jones vector notation to write its fluctuating electrical field at a time $t$ and position $z$ in the form

$$E(t, r) = \left( \begin{array}{c} A_1(t, z) \\ A_2(t, z) \end{array} \right) F(x, y) e^{i\beta_0 (z - \omega_0 t)},$$  

(1)

where $\omega_0$ is the carrier frequency and $\beta_0$ is an effective propagation constant at this frequency of the single mode supported by the fiber with the spatial profile $F(x, y)$. Physically, $A_1(t, z)$ and $A_2(t, z)$ are the slowly varying amplitudes of the two mutually orthogonal polarization components of the field $E(t, r)$. The correlation properties of the pulse can then be characterized by a second-order correlation tensor, defined as [1]

$$\Gamma_{jl}(t_1, t_2, z) = \left( \begin{array}{c} A_j(t_1, z) \end{array} \right) A_l(t_2, z),$$  

(2)

where $j$ and $l$ take on values $x$ or $y$ and the angle brackets denote ensemble averaging.

In the linear propagation regime, the evolution of any frequency component of the two polarization modes is governed by $B_j(\omega, z) = B_j(\omega, 0) \exp[i(\beta_j(\omega) - \beta_0)z]$, where $B_j$ represents the Fourier transform of $A_j$, and the propagation constants $\beta_j$ for the two modes are different because of the birefringence. Expanding $\beta_j(\omega)$ in a Taylor series around the carrier frequency $\omega_0$ and taking the inverse Fourier transform, $A_j(t, z)$ is found to satisfy the wave equation [6]

$$\frac{\partial A_j}{\partial z} + \beta_j A_j + \frac{i}{2} \beta_{j2} \frac{\partial^2 A_j}{\partial z^2} = i(\beta_{0j} - \beta_0) A_j,$$

(3)

where $\beta_{j2} = (d^m \beta_j / d\omega^m)_{\omega=\omega_0}$. Physically, $\beta_{0j} = 1/\nu_{j1}$, where $\nu_{j1}$ is the group velocity, and $\beta_{j2}$ accounts for the GVD. For most fibers, the GVD is nearly the same for the two polarization modes, i.e., $\beta_{j1} = \beta_{j2} = \beta_j$, but their group velocities differ because of fiber birefringence.

To simplify Eq. (3), we introduce new variables as

$$\xi = \omega_0 \beta_2 z, \quad \tau = t - z \beta_{11}, \quad \tilde{A}_j(\tau, \xi) = A_j(t, z) \exp[-i(\beta_{0j} - \beta_0)z],$$

(4)

which results in the simple wave equation

$$2i\omega_0 \frac{\partial \tilde{A}_j}{\partial \xi} = \frac{i^2 \omega_0}{2\pi^2} \tilde{A}_j(\tau, \xi) \exp[-i(\beta_{0j} - \beta_0)z],$$

(5)

Solving this equation with a standard Fourier–transform technique, we find that

$$\tilde{A}_j(\xi, \tau) = \sqrt{\frac{i\omega_0}{2\pi^2}} \int_{-\infty}^{\infty} \tilde{A}_j(\tau) \exp\left[-\frac{i\omega_0}{2\xi}(\tau - \tau_0)^2\right] d\tau,$$

(6)

where $\tilde{A}_j(\tau)$ is the amplitude of the $j$th polarization component in the source plane $z=0$. It follows from Eqs. (2) and (4) that

$$\Gamma_{jl}(t_1, t_2, z) = \tilde{\Gamma}_{jl}(\tau_1, \tau_2, \xi) e^{i(\beta_{0j} - \beta_0)z},$$

(7)

$$\tilde{\Gamma}_{jl}(\tau_1, \tau_2, \xi) = \langle \tilde{A}_j(\tau_1, \xi) \tilde{A}_l(\tau_2, \xi) \rangle,$$

(8)

where $\tau_1 = t_1 - z \beta_{11}$ and $\tau_2 = t_2 - z \beta_{11}$.

To determine the second-order correlation tensor in any transverse plane $z = \text{const} > 0$, we focus on a particular GSM source that generates the field with the correlation tensor of the form

$$\tilde{\Gamma}_{jl}(0, 0, \xi) = A^2 \delta_{jl} \exp\left[-\frac{(\tau_1^2 + \tau_2^2)_{\xi}}{4\sigma^2_{\xi}} \right] \exp\left[-\frac{(\tau_1 - \tau_2)^2}{2\sigma_{\tau}^2} \right].$$

(9)

The model of Eq. (9) describes rather well the output of the nonstationary statistical light sources that can be realized by temporally modulating statistically stationary sources with Gaussian spectra such as light-emitting diodes [24] and quasi-white light sources that mimic thermal radiation [25]. Here the initial pulse width $\sigma_\tau$ is assumed to be the same for both polarization modes, and $\sigma_{\xi}$ stands for the correlation time of the $j$th polarization mode. After some algebra, it follows from Eqs. (6), (8), and (9) that

$$\tilde{\Gamma}_{jl}(0, 0, \xi) = \frac{A^2 \delta_{jl}}{\Delta_j(\xi)} \exp\left[-\frac{\gamma_j \tau_1^2 + \gamma_j \tau_2^2 - 2\beta_j \tau_1 \tau_2}{\sigma_{\xi}^2 \Delta_j^2(\xi)} \right],$$

(10)

where we have introduced the following notation:

$$\frac{1}{\rho_j^2} = \frac{1}{4\sigma_{\xi}^2} + \frac{1}{\sigma_{\tau}^2}, \quad \gamma_j = \alpha_j - \frac{i\xi}{2\omega_0},$$

(11)

$$\alpha_j = \frac{\omega_j}{4(a_j^2 - b_j^2)}, \quad \beta_j = \frac{b_j}{4(a_j^2 - b_j^2)},$$

(12)

$$\alpha_j = \frac{1}{4\sigma_{\xi}^2} + \frac{1}{2\sigma_{\tau}^2}, \quad b_j = \frac{1}{2\sigma_{\tau}^2},$$

(13)

$$\Delta_j(\xi) = \sqrt{1 + \frac{\rho_j^2}{\sigma_{\xi}^2} \exp\left(i\rho_j^2\xi\right)}. $$

(14)

Equation (10) shows how the coherence properties of the pulse change with propagation. The ensemble-averaged $2 \times 2$ matrix $J(t, z)$, characterizing the polarization properties of the pulse at some point $(t, z)$, is referred to as the coherency tensor with its matrix elements defined as $J_{jl}(t, z) = \langle A_j(t, z) A_l(t, z) \rangle$. The degree of polarization can be expressed in terms of the coherency tensor as [1]

$$P(t, z) = \left(1 - \frac{4 \det[J(t, z)]}{(tr[J(t, z)])^2} \right)^{1/2},$$

(15)

where det and tr denote the determinant and the trace of a matrix, respectively. The coherency tensor is related to the second-order correlation tensor by the expression

$$J_{jl}(t, z) = \tilde{\Gamma}_{jl}(0, 0, \xi) e^{i(\beta_{0j} - \beta_0)z}. $$

(16)

Consider first the polarization properties in the source plane $z=0$. We assume that the $x$ and $y$ polarization modes have the same intensities and are uncorrelated in the source plane, in agreement with Eq. (9). This assumption results in the coherency tensor at $z=0$ in the form...
The degree of polarization also demonstrated numerically for a general case when both birefringence and polarization-dependent coherence properties are considered. For the numerical calculations, we employ $\sigma_z=10$ ps and $\beta_2=17.8$ ps$^2$/km, which are typical values for picosecond pulses propagating in silica-glass fibers [6].

We first consider a polarization-independent partially coherent light source, i.e., $\sigma_x=\sigma_y=\sigma_z$. In this case, the general expression for the degree of polarization (21) can be simplified as follows:

$$P(\tau,z) = \tanh \left[ \frac{\tau \delta \epsilon}{\sigma_f^2 \Lambda^2(z)} \right].$$

(24)

It follows immediately from Eq. (24) that at $\tau=0$, we have $P(0,z)=0$, indicating that the light remains completely unpolarized at the center of the pulse. This is expected since the intensity profiles of the two polarization modes have the same widths at the source, and the walk-off due to birefringence will cause the intensities of the two modes to differ at any $\tau$ except at $\tau=0$. For any $\tau \neq 0$, such as $\tau=\delta \epsilon$, we transform the field from a fixed coordinate system to a coordinate system moving with the group velocity of the $x$ mode.

The degree of polarization $P$ as a function of the propagation distance $z$ is displayed in Fig. 1 for $\sigma_z=100, 20, 5$, and 1 ps. The solid, dotted–dashed, and dotted curves correspond to $\delta=2.6$, 0.52, and 0.13 ps/km, respectively. As seen in Fig. 1, the degree of polarization steadily increases with the propagation distance and approaches its asymptotic value regardless of the choice of $\sigma_z$ and $\delta$. The asymptotic value of $P$ depends on the choice of $\sigma_z$ and $\delta$ and is given by the expression

$$P_\infty = \tanh \left[ \frac{\sigma_f^2}{4 \sigma_z^2} \left( \frac{1}{\sigma_z^2} + \frac{1}{\sigma_f^2} \right)^{-1} \right].$$

(25)

In particular, for nearly coherent light ($\sigma_z<\sigma_f$), the light can become completely polarized if birefringence is also large enough, as evident from Figs. 1(a) and 1(b). This is expected since the walk-off effect dominates when the pulse spreading is relatively small, resulting eventually in complete temporal separation of the two polarization modes from each other. On the other hand, when the light is nearly incoherent, the pulse spreading can be so pronounced that the walk-off effect becomes less prominent. As a consequence, the intensity profiles of the two polarization modes nearly overlap despite the walk-off, and the

$$J(t,z=0) = I_0(t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

(17)

where $I_0(t)=A^2 \exp(-t^2/2\sigma_z^2)$. It follows from Eq. (15) and Eq. (17) that $P=0$, i.e., the light in the source plane is completely unpolarized.

To find changes in the degree of polarization at a distance $z$, we first calculate the coherency tensor at any point $(t,z)$ according to Eqs. (10), (16), and (17), which results in

$$J(t,z) = \begin{bmatrix} I_x(t,z) & 0 \\ 0 & I_y(t,z) \end{bmatrix},$$

(18)

where the individual mode intensities are given by

$$I_j(t,z) = \frac{A^2}{\Lambda_j(z)} \exp \left[ -\frac{(t-z \beta_j)^2}{2\sigma_j^2 \Lambda_j^2(z)} \right],$$

(19)

and the temporal broadening factor of each polarization component depends on the GVD parameter $\beta_j$ as

$$\Lambda_j(z) = \sqrt{1 + \left( \frac{\beta_j \epsilon}{2\sigma_j^2} \right)^2 + \left( \frac{\beta_j \epsilon}{\sigma_j \sigma_{\epsilon j}} \right)^2}.$$  

(20)

As expressed in Eq. (18), the $x$ and $y$ polarization modes remain uncorrelated on propagation of the pulse along the fiber, but their intensities become different. The temporal width of these two components becomes different because of the GVD and polarization-dependent correlation times. The two modes also walk off spatially because of their different group velocities as a result of fiber birefringence.

### 3. PROPAGATION-INDUCED CHANGES IN THE DEGREE OF POLARIZATION

To simplify the expression of the degree of polarization, we substitute Eqs. (18)–(20) into Eq. (15). It is convenient to transform the field $E(t,\mathbf{x})$ from the fixed coordinate system to a coordinate system moving with the average group velocity $v_g=1/\beta$, where $\beta=(\beta_x+\beta_y)/2$, so that the degree of polarization can be expressed as a function of the retarded time $\tau=t-\beta z$ and the propagation distance $z$ as

$$P(\tau,z) = \frac{|I_x(\tau,z) - I_y(\tau,z)|}{I_x(\tau,z) + I_y(\tau,z)},$$

(21)

where

$$I_j(\tau,z) = \frac{A^2}{\Lambda_j(z)} \exp \left[ -\frac{(\tau-\beta z \epsilon)^2}{2\sigma_j^2 \Lambda_j^2(z)} \right].$$

(22)

Here $+$ and $-$ signs correspond to the $x$- and $y$-polarized modes, respectively, and $\delta=(\beta_y-\beta_x)/2$ represents the extent of group-velocity mismatch. It follows from Eq. (22) that the different dynamics of the intensity profiles of the two polarization modes can be caused by the competition between the walk-off effect and the polarization-dependent pulse spreading. The walk-off effect results from the fiber birefringence, while the widths of the intensity profiles increase at different rates because of different correlation times of the two polarization components at the source. Specifically, according to Eq. (22), the width of the $j$th polarization mode profile at a distance $z$ from the source can be expressed as

$$\tau_{\epsilon j}(z) = \sqrt{\frac{\sigma_j^2 + \beta_j^2 \epsilon^2 + \frac{1}{\sigma_j^2}}{\frac{4}{\sigma_z^2} + \frac{1}{\sigma_f^2}}}.$$  

(23)

which clearly depends on both $\sigma_j$ and the correlation time $\sigma_{\epsilon j}$ of the corresponding polarization component.

We now analyze, both analytically and numerically, the propagation of the degree of polarization in special cases in which either coherence properties of the light in the source plane are assumed polarization independent, or fiber birefringence is ignored. The evolution of the degree of polarization is also demonstrated numerically for a general case when both birefringence and polarization-dependent coherence properties are considered. For the numerical calculations, we employ $\sigma_z=10$ ps and $\beta_2=17.8$ ps$^2$/km, which are typical values for picosecond pulses propagating in silica-glass fibers [6].

The degree of polarization $P$ as a function of the propagation distance $z$ is displayed in Fig. 1 for $\sigma_z=100, 20, 5$, and 1 ps. The solid, dotted–dashed, and dotted curves correspond to $\delta=2.6$, 0.52, and 0.13 ps/km, respectively. As seen in Fig. 1, the degree of polarization steadily increases with the propagation distance and approaches its asymptotic value regardless of the choice of $\sigma_z$ and $\delta$. The asymptotic value of $P$ depends on the choice of $\sigma_z$ and $\delta$ and is given by the expression

$$P_\infty = \tanh \left[ \frac{\sigma_f^2}{4 \sigma_z^2} \left( \frac{1}{\sigma_z^2} + \frac{1}{\sigma_f^2} \right)^{-1} \right].$$

(25)

In particular, for nearly coherent light ($\sigma_z<\sigma_f$), the light can become completely polarized if birefringence is also large enough, as evident from Figs. 1(a) and 1(b). This is expected since the walk-off effect dominates when the pulse spreading is relatively small, resulting eventually in complete temporal separation of the two polarization modes from each other. On the other hand, when the light is nearly incoherent, the pulse spreading can be so pronounced that the walk-off effect becomes less prominent. As a consequence, the intensity profiles of the two polarization modes nearly overlap despite the walk-off, and the
pulse remains almost completely unpolarized as is evident from Fig. 1(d).

The extent of variations of the degree of polarization across the pulse for a fixed $z$ is shown in Fig. 2, where we chose $\delta=0.26$ ps/km, and $\sigma_c$ takes the values 1, 5, 20, and 100 ps, the same four values used in Fig. 1. The solid, dotted–dashed, and dotted curves correspond to $z=2$, 6, and 12 km, respectively. As expected, the degree of polarization remains zero at the center of the pulse and increases symmetrically on the two sides of pulse center, irrespective of the choice of $\sigma_c$ and $z$. Note that the light remains nearly unpolarized across the entire pulse when the coherence time of the input pulse is much shorter than the pulse width ($\sigma_c \ll \sigma_t$).

We analyze next the impact of the polarization-dependent coherence properties on the change of polarization as light propagates along the fiber with or without birefringence. For an ideal fiber with no birefringence, the
degree of polarization is affected by the pulse spreading, and this can be seen from Eqs. (20)–(22) by setting $\delta=0$. According to Eq. (23), the widths of the two polarization components increase at different rates when $\sigma_{x} \neq \sigma_{y}$. As a consequence, the difference in the widths of intensity profiles for the two polarization modes also increases with the propagation distance along the fiber. It can be inferred from Eq. (22) that at a certain propagation distance $z$, one mode has larger width but lower intensity than the other, which induces changes in the degree of polarization.

Variations in the degree of polarization across the pulse are calculated for a fiber with negligible birefringence and partially coherent light with $\sigma_{x}=12$ ps and $\sigma_{y}=16$ ps. The results are exhibited in Fig. 3(a) in which the solid, dotted–dashed, and dotted curves correspond to $z=5$, 20, and 40 km, respectively; they correspond to $\tau=0$, 20, and 80 ps, respectively, in parts (c) and (d). Variations in the degree of polarization across the pulse are calculated for a fiber with negligible birefringence and partially coherent light with $\sigma_{x}=12$ ps and $\sigma_{y}=16$ ps. The results are exhibited in Fig. 3(a) in which the solid, dotted–dashed, and dotted curves correspond to $z=5$, 20, and 40 km, respectively; they correspond to $\tau=0$, 20, and 80 ps, respectively, in parts (c) and (d).

Fig. 3. Variations of the degree of polarization across the pulse for (a) $\delta=0$ and (b) $\delta=0.26$ ps/km. The degree of polarization as a function of $z$ for (c) $\delta=0$ and (d) $\delta=0.26$ ps/km for partially coherent light with $\sigma_{x}=12$ ps and $\sigma_{y}=16$ ps. In parts (a) and (b), the solid, dotted–dashed, and dotted curves correspond to $z=5$, 20, and 40 km, respectively; they correspond to $\tau=0$, 20, and 80 ps, respectively, in parts (c) and (d).

Fig. 4. Normalized intensity of the whole pulse (solid curve) and of the two polarization modes (dashed curves) as a function of $\tau$ at $z=10$ km. The degree of polarization $P$ is also plotted for comparison (dotted curve).
pulse for a birefringent fiber by choosing $\delta=0.26$ ps/km. The result is displayed in Fig. 3(b). Comparing parts (a) and (b) of Fig. 3, we conclude that the walk-off effect resulting from fiber birefringence does not break the symmetry of the degree-of-polarization profile. Rather, the center of symmetry is shifted away from the pulse center. The degree of polarization as a function of $z$ is also calculated with or without birefringence, and the results are shown in parts (c) and (d) of Fig. 3 using $\delta=0$ and $\delta=0.26$ ps/km, respectively. The solid, dotted–dashed, and dotted curves correspond to three temporal positions across the pulse, $\tau=0$, 20, and 80 ps, respectively. Contrasting with the case of a source with polarization-independent coherence properties shown in Fig. 1, the degree of polarization in this case does not necessarily increase monotonically along the fiber, and it does not remain zero at the center of the pulse.

To illustrate the effects of the walk-off and of polarization-dependent spreading on the pulse intensity, we plot in Fig. 4 the normalized intensities of the whole pulse and the two polarization modes as functions of $\tau$ at $z=10$ km. The degree of polarization $P$ across the entire pulse is also displayed for comparison. Interestingly, the total pulse intensity remains symmetric about the pulse center despite the presence of both the walk-off effect and polarization-dependent spreading.

4. CONCLUSIONS

We have studied the behavior of the degree of polarization of partially coherent pulses propagating along dispersive and birefringent optical fibers in the linear regime. We have shown that the polarization dynamics is determined by a subtle interplay of two factors: (i) the coherence properties of the input pulse at the source plane and (ii) the spatial walk-off of the two mutually orthogonal polarization components of the pulse caused by the fiber birefringence. Consequently, even when the coherence properties of the two polarization components of the pulse in the source plane are the same, the degree of polarization changes along the fiber because of the group-velocity mismatch. This circumstance makes polarization changes on pulse propagation in fibers qualitatively and quantitatively different from those induced by partially coherent beam propagation in free space [8]. Thus the space–time analogy existing for partially coherent scalar pulses propagating in linear dispersive media and partially coherent scalar beams propagating in free space [26] cannot be fully extended to their electromagnetic counterparts. In future work, we plan to take into account the influence of the fiber nonlinearity and random birefringence on the state of polarization of intense statistical pulses propagating in realistic optical fibers.

REFERENCES