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'Slow' and 'fast' light in resonator-coupled waveguides

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Abstract. We describe a device constructed of a sequence of microresonators coupled to an optical waveguide. The influence of these resonators is to enhance nonlinearities and to induce strong dispersive effects, leading to exotic optical properties including slow and superluminal group velocities of propagation.

In recent years there has been a flurry of activity aimed at the development of techniques that can lead to a significant modification of the group velocity of propagation of a light pulse through a material medium [1]. Proposed applications of these procedures include the development of optical delay lines [2] and the 'storage' of light pulses [3, 4] with perhaps implications for the field of quantum information. Most of this research has made use of the response of resonant media [5] and much of it has made use of the concept of electromagnetically induced transparency [3, 6].

In this contribution, we describe an alternative procedure for the propagation of slow light based on inducing large dispersive effects in optical waveguides by coupling the waveguide to an array of optical resonators. A typical device of this sort, which we refer to [7, 8] as a side-coupled integrated spaced sequence of resonators (SCISSOR), is shown in figure 1. The resonators can be of arbitrary design, although in our experimental work we are concentrating on resonators in the form of ring waveguides or of a whispering gallery mode [9] of dielectric discs. Alternatively, the resonators could be dielectric spheres coupled to an optical fibre; the excitation of the resonances of such spheres has been observed previously [10]. Since the light field effectively circulates many times in each resonator before passing to the next, the group velocity of propagation of a pulse of light through such a structure is significantly reduced. In addition, the phase shift experienced by the light wave in interacting with each resonator depends sensitively on its detuning from the cavity resonance, and thus this structure produces large and controllable dispersion. Moreover, if the resonator is constructed of a material that displays a nonlinear optical response, the nonlinear phase shift acquired by a light wave in interacting with the resonator scales as the square of the finesse of the resonator [11]; thus such a structure displays an enhanced nonlinear optical response. For these reasons, devices of this sort may prove extremely useful for many applications in modern photonics. Related approaches offering similar promise, but possessing photonic bandgaps (which are necessarily attenuating)

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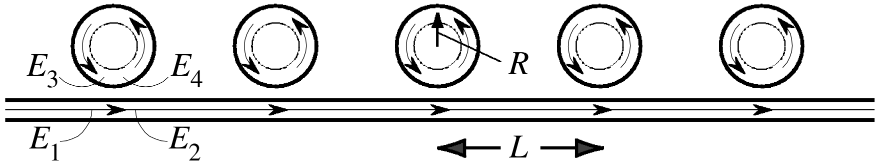

Figure 1. A SCISSOR.

are Bragg gratings [12, 13], photonic crystal structures [14] and coupled resonator optical waveguide structures recently proposed [15]. The SCISSOR structure is unique in that it can induce strong delay and dispersion while greatly enhancing weak intrinsic nonlinearities without introducing a photonic bandgap.

In order to understand the optical properties of a SCISSOR device, let us first recall the treatment of the optical properties of a single resonator coupled to an optical waveguide. We describe the coupling of light into and out of the resonator in terms of effective reflection and transmission coefficients r and t according to

$$\begin{pmatrix} \widetilde{E}_4(\omega) \\ \widetilde{E}_2(\omega) \end{pmatrix} = \begin{pmatrix} r & \mathrm{i}t \\ \mathrm{i}t & r \end{pmatrix} \begin{pmatrix} \widetilde{E}_3(\omega) \\ \widetilde{E}_1(\omega) \end{pmatrix}, \tag{1}$$

where we assume that $r^2 + t^2 = 1$ and where the various field amplitudes are defined as in figure 1. Each optical resonator of the SCISSOR structure behaves in many ways like a Fabry–Pérot cavity but with the crucial distinction that it has only a single output port and thus (ignoring for the present, attenuation effects) is an ‘all-pass filter’ which simply impresses a phase shift Φ on to the transmitted field. To calculate this phase shift, we first note that the *internal* or *single-pass* phase shift ϕ imposed on a wave circulating within the resonator is given by $\phi = (\omega - \omega_{\mathrm{R}})T$, where ω_{R} is one of the resonance frequencies of the resonator and T is the circumferential transit time $2\pi nR/c$. Since $\widetilde{E}_3(\omega) = \exp[\mathrm{i}\phi(\omega)]\,\widetilde{E}_4(\omega)$ and $\widetilde{E}_2(\omega) = \exp[\mathrm{i}\Phi(\omega)]\,\widetilde{E}_1(\omega)$ we find through use of equation (1) that the total phase shift Φ is given by

$$\Phi = \pi + \phi + 2\arctan\left(\frac{r\sin\phi}{1 - r\cos\phi}\right). \tag{2}$$

This phase shift Φ that the light wave experiences in interacting with each resonator modifies the propagation of light through a sequence of such resonators. In particular, the effective propagation constant becomes $k_{\mathrm{eff}} = n\omega/c + \Phi/L$, where L is the spacing between resonators. Near each cavity resonance, the phase Φ varies rapidly with frequency, leading to a greatly reduced group velocity. In particular, one finds that the group index for a pulse whose carrier wave acquires an internal phase shift ϕ_0 is given by

$$\begin{aligned} n_{\mathrm{g}} &\equiv \frac{c}{v_{\mathrm{g}}} \\ &= c\,k'_{\mathrm{eff}} \\ &= c\,\frac{\mathrm{d}k_{\mathrm{eff}}}{\mathrm{d}\omega} = n + \frac{c}{L}\frac{\mathrm{d}\Phi}{\mathrm{d}\omega} \\ &= n\left(1 + \frac{2\pi R}{L}\frac{1 - r^2}{1 - 2r\cos\phi_0 + r^2}\right). \end{aligned} \tag{3}$$

The group index takes its maximum value when the optical wave is tuned to a cavity resonance ($\phi = 0$), leading to an equation for the group index that can be expressed in any of the following forms:

$$n_g = n\left(1 + \frac{2\pi R}{L}\frac{1+r}{1-r}\right) = n\left(1 + \frac{2\pi R}{L}\mathcal{B}_0\right) = n\left(1 + \frac{4R}{L}\mathcal{F}\right) \tag{4}$$

where $\mathcal{B}_0 = (1+r)/(1-r)$ can readily be shown to express the on-resonance build-up factor of light within the resonator (i.e., $\mathcal{B} \equiv |E_4|^2/|E_1|^2$) and where $\mathcal{F} = (\pi/2)\mathcal{B}_0$ is the finesse of the resonator. The frequency dependence of the build-up factor $\mathcal{B}$, the resonator contribution to propagation constant and the group index are shown in figure 2.

While the steep slope of the dispersion-relation curve near resonance is responsible for the reduced group velocity, the transition from the flat sections of the dispersion curve to the steep section is necessarily curved and introduces group velocity dispersion (GVD). On resonance, the lowest-order GVD parameter k''_{eff} (and all other even orders) is zero. However, the distance over which pulses can propagate is limited ultimately by broadening induced by third-order dispersion. We find that it is better to propagate off resonance and to sacrifice some enhancement in order to gain much in terms of the maximum possible propagation distance. Specifically, by tuning slightly above resonance (such that $\omega = \omega_R + \pi/\mathcal{F}T3^{1/2}$), third-order dispersion can be eliminated. The lowest-order

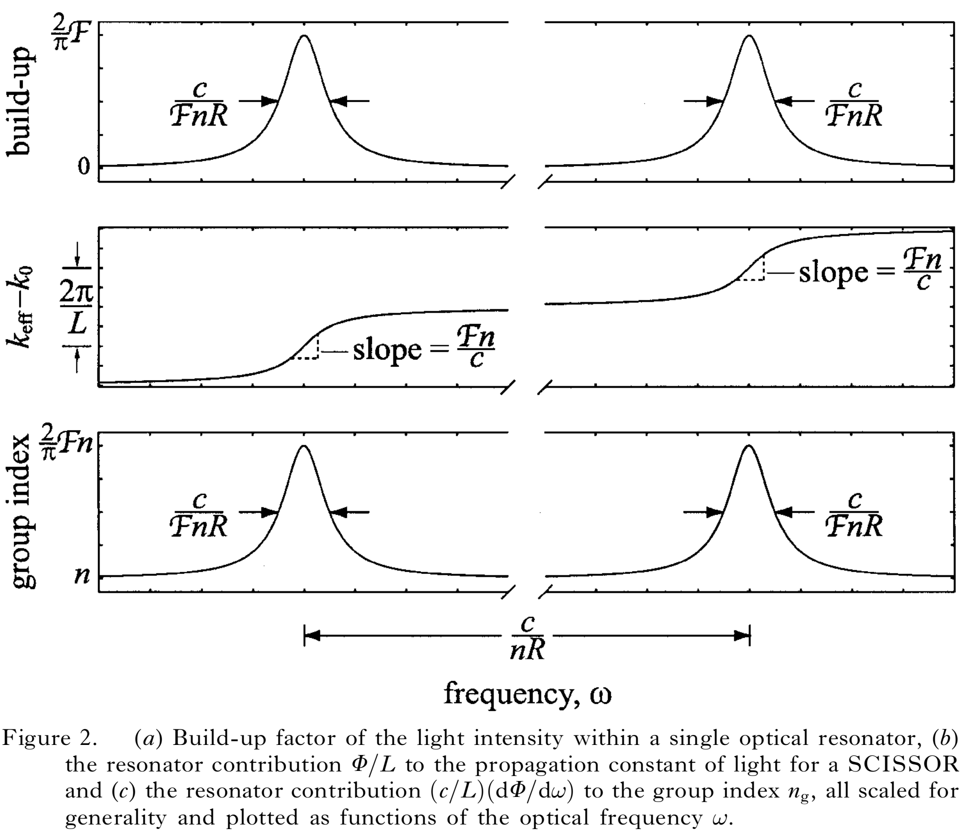

Figure 2. (*a*) Build-up factor of the light intensity within a single optical resonator, (*b*) the resonator contribution Φ/L to the propagation constant of light for a SCISSOR and (*c*) the resonator contribution $(c/L)(\mathrm{d}\Phi/\mathrm{d}\omega)$ to the group index n_g, all scaled for generality and plotted as functions of the optical frequency ω.

dispersion that is introduced by operating off resonance can be compensated for by the enhanced nonlinear response of the structure. Under proper conditions, the negative lowest-order GVD occurring at this operating point can be precisely balanced by the nonlinearity to form a SCISSOR soliton [7, 8]. The enhanced-nonlinearity coefficient $\gamma_{\rm eff}$ is obtained from the build-up and the group velocity reduction factors as

$$\begin{aligned}\gamma_{\rm eff} &\equiv \frac{1}{L}\frac{\mathrm{d}\Phi}{\mathrm{d}|\widetilde{E}_1|^2} \\ &= \frac{1}{L}\frac{\mathrm{d}\Phi}{\mathrm{d}\phi}\frac{\mathrm{d}\phi}{\mathrm{d}|\widetilde{E}_3|^2}\frac{\mathrm{d}|\widetilde{E}_3|^2}{\mathrm{d}|\widetilde{E}_1|^2} \\ &= \frac{\gamma 2\pi R}{L}\left(\frac{1-r^2}{1-2r\cos\phi_0+r^2}\right)^2 \\ &\xrightarrow[\phi_0=0,\, r\approx 1]{} \gamma\frac{8R}{\pi L}\mathcal{F}^2.\end{aligned} \tag{5}$$

Figure 3 shows the frequency dependences of the lowest-order GVD $k''_{\rm eff}$ and enhanced nonlinearity $\gamma_{\rm eff}$ and also indicates the optimum detuning for soliton propagation.

Figure 4 compares three approaches to attempting to propagate slow light in a SCISSOR with a group velocity of approximately$(c/n)/100$. In figure 4 (*a*), a weak pulse tuned to resonance is greatly delayed but broadens and acquires ripples associated with negative third-order dispersion. In figure 4 (*b*), the pulse frequency is tuned above resonance to the extremum of the lowest-order GVD†. At this frequency, the third-order GVD necessarily vanishes, and we see that pulse distortion of the sort shown in figure 4 (*a*) is prevented from occurring. However, the pulse broadens considerably as a result of non-vanishing second-order (lowest-order) dispersion. In figure 4 (*c*), the same pulse but with a peak power corresponding to that of the fundamental SCISSOR soliton is seen to propagate with a preserved pulse shape. The group velocity reduction in this case is 75× as opposed to 100× in figure 4 (*a*), but the high fidelity of pulse propagation makes this strategy appear to be superior.

The analysis given above has assumed that the resonators are non-attenuating. In practice, microresonators suffer from intrinsic absorption (both linear and nonlinear), bending loss and scattering loss due to surface imperfections which lower the transmission and build-up factor. Of course, gain may be added to the system, if possible, to offset these losses. The net attenuation near resonance is increased in proportion to the resonator finesse. In fact, if the resonator coupling strength and round-trip loss are exactly equal, the resonator is said to be critically coupled and the net loss is complete, that is the transmission is zero. Although the

† The simulations used to study pulse evolution in a sequence of waveguide-coupled resonators were carried out using an iterative method in which each iteration consisted of linear and nonlinear phase accumulation during one round trip within the resonator followed by interference at the coupler. Traditional beam or pulse propagation split-step Fourier methods are unnecessary as structural dispersion is more readily treated in the time domain. The values of $k'_{\rm eff}$ and $\gamma_{\rm eff}$ are lowered by factors of 3/4 and 9/16 respectively from their given maximum values when operating at the anomalous GVD maximum.

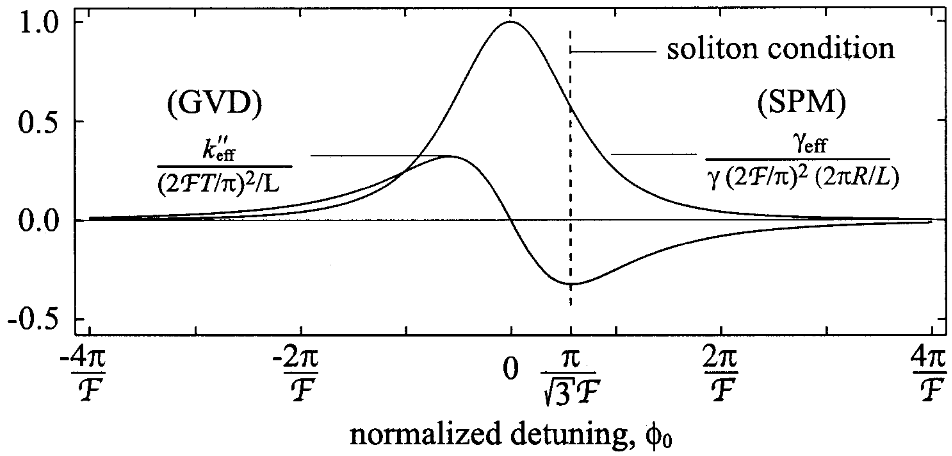

Figure 3. Plot of the frequency dependence of the effective nonlinear self-phase modulation (SPM) coefficent $\gamma_{\rm eff}$ and the GVD parameter $k''_{\rm eff}$. Note that the sign of the GVD parameter $k''_{\rm eff}$ changes with the sign of the detuning from resonance. The curves are normalized for generality. The broken line indicates the optimum detuning for soliton propagation and corresponds to the conditions used in figure 4 (*c*).

above analysis assumed that the resonators are lossless, in fact the qualitative features of this analysis are still valid provided that the coupling strength is greater than the round trip loss. Such a resonator is said to be overcoupled. Low-loss propagation through a SCISSOR constructed from N resonators can be ensured if the single-pass attenuation satisfies $\alpha 2\pi R \ll 1/N\mathcal{F}$, where α is the absorption coefficient of the resonator material. The dispersion relation becomes steeper near resonance with increasing attenuation in the overcoupled regime, implying an even greater group velocity reduction. Of course, this reduction in group velocity is obtained at the price of reduced output power.

Even more interestingly, we find that lossy resonators forming a SCISSOR structure can be implemented to propagate light superluminally [16]. If the coupling strength is chosen to be weaker than the round-trip loss, the resonator is said to be undercoupled. The dispersion relation for the SCISSOR in this regime is qualitatively different from that found in the overcoupled regime. Figure 5 (*a*) displays the transmission for a single resonator in the three regimes. In addition, figure 5 (*b*) displays the resonator contribution to the dispersion relation for all three regimes and displays the reversal of sign near resonance. This negative slope implies that a pulse exiting from a resonator will emerge with its centre advanced in time with respect to the incident pulse. Causality is maintained because the discrete impulse response of the resonator does not possess any advanced impulses. A finite bandwidth pulse displays superluminal propagation but quickly attenuates in a multiresonator undercoupled SCISSOR. Gain (assumed flat across the pulse bandwidth) may be incorporated into the straight waveguide to offset the losses associated with the undercoupled configuration. The group index near resonance in the undercoupled regime is given by

$$n_g = n\left(1 - \frac{2\pi R}{L}\frac{a(1-r^2)}{(r-a)(1-ra)}\right), \qquad (6)$$

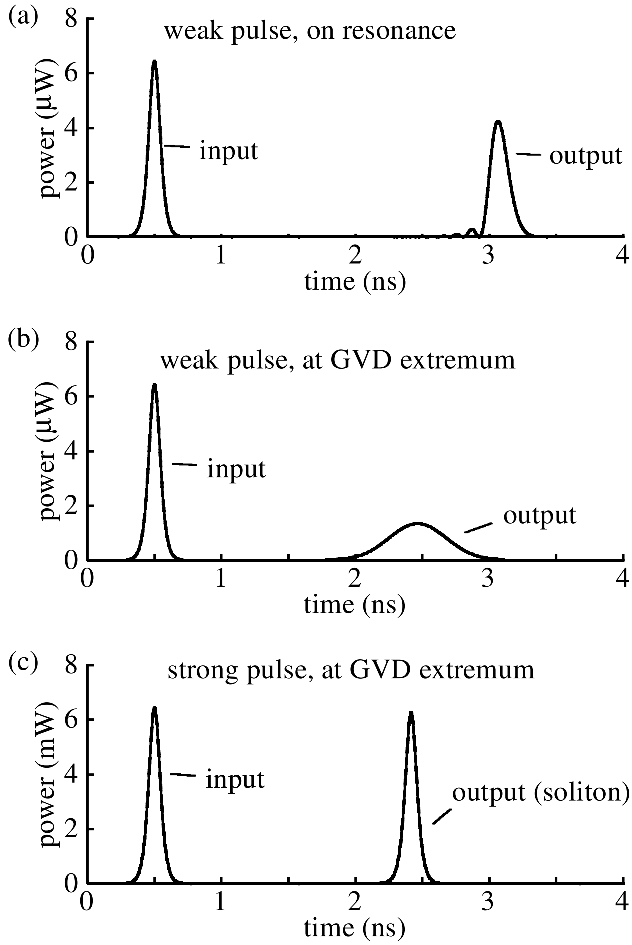

Figure 4. Numerical results showing the advantage of using optical nonlinearity for propagating slow light through a SCISSOR structure. The SCISSOR consists of 100 resonators of 10 μm diameter spaced by 10π μm with $r = 0.98$ corresponding to a group velocity reduction of 100×. (*a*) A weak 100 ps resonant pulse propagates at a group velocity of $(c/n)/100$ through the SCISSOR and is corrupted by resonator-induced third-order dispersion. (*b*) The same pulse, but with its carrier frequency tuned to the extremum of the lowest-order GVD, propagates with a group velocity of $(c/n)/75$ but is greatly broadened. (*c*) A 100 ps pulse of 6.4 mW peak power, tuned to the GVD extremum propagates as the fundamental SCISSOR soliton with a group velocity of $(c/n)/75$ and is well preserved. Here we assumed parameters typical of a GaAs or chalcogenide-glass-based waveguiding structure ($\gamma = 60\ \mathrm{m}^{-1}\ \mathrm{W}^{-1}$).

where $a = \exp(-\alpha\pi R)$. Figure 5 (*c*) demonstrates superluminal propagation of a linear resonant pulse through undercoupled resonators situated near an amplifying waveguide. The exiting pulse experiences a negative time delay or, equivalently, a time advance. Moreover, the theory also predicts superluminal propagation for a SCISSOR in which each resonator is constructed of an amplifying medium in which the round trip gain is greater than the coupling strength; however, our numerical simulations suggest that propagation is highly unstable in this regime.

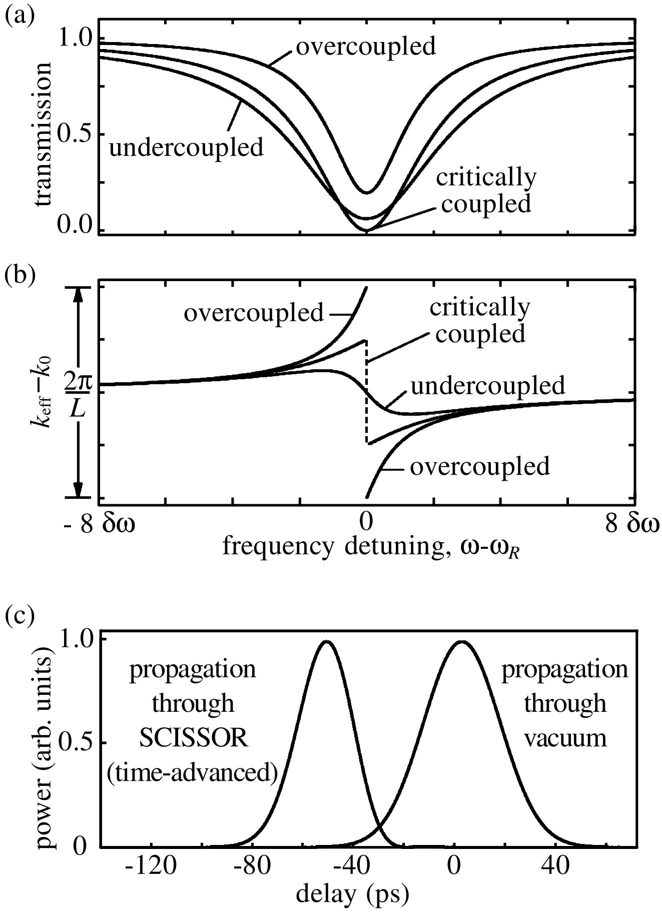

Figure 5. (*a*) The transmission for a single resonator with $r = 0.9$ in the overcoupled ($a = 0.96$) critically coupled ($a = 0.9$), and undercoupled ($a = 0.84$) regimes. (*b*) The dispersion relation for the same three cases. Note the change in the sign of the slope of the curve near resonance in the undercoupled case. In the critically coupled case, the curve undergoes a π/L phase jump at the cavity resonance frequency (where the transmission is zero). In the overcoupled case, the second half of curve has been cut and displaced downwards by $2\pi/L$ for generality of display. The frequency axis is labelled in units of $\delta\omega$, which correspond to the resonator linewidth in the limit of negligible atttenuation. (*c*) Numerical simulation demonstrating superluminal propagation of a 36 ps pulse through a SCISSOR structure composed of 20 lossy undercoupled resonators of 10 μm diameter, spaced by 10π μm. Gain has been added to the straight waveguide section to maintain pulse power.

In conclusion, we have demonstrated that waveguides modified with side-coupled resonators can produce 'slow' or 'fast' light propagation. Fabrication methods for producing microresonators are rapidly advancing [17–19] and it is expected that microresonators will serve as building blocks for forming micro-resonator arrays for integrated photonics [20]. Dispersive effects resulting from the frequency dependence of resonant peaks ultimately limit the range over which slow-light propagation may be achieved in a SCISSOR structure. We have demonstrated a better way to propagate slow light involving a balance between resonator induced GVD and resonator enhanced nolinearity employing soliton robustness. Finally, in practice, microresonators are typically lossy; we have shown that this loss can be implemented to study superluminal light propagation.

Acknowledgments

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