Correlated amplification of light

Martti Kauranen and Robert W. Boyd
The Institute of Optics, University of Rochester, Rochester, New York 14627
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We show that it is possible to amplify a beam of light in such a way that less noise is added to the beam than would be added by a quantum-noise-limited amplifier. This result can be interpreted in terms of a filtering mechanism that makes it possible to reduce the effective noise bandwidth of the amplifier.

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I. INTRODUCTION

The use of nonlinear optical interactions in the generation of squeezed states of light [1] has been established in several experiments. For a squeezed state of light, the quantum-mechanical fluctuations in one of the field variables are smaller than those for a coherent state of the same average intensity; thus the noise associated with the measurement of this variable is below the conventional shot-noise limit. Quantum-mechanical properties of the electromagnetic field also limit the noise performance of optical amplifiers [2,3]. In particular, for the case in which the input into a quantum-noise-limited amplifier is in a coherent state, the variance of the photon-number fluctuations of the output field is given by the quantum-mechanical amplifier-noise limit

$$\langle (\Delta \hat{n}_{\text{out}})^2 \rangle_{\text{ANL}} = G(2G-1)\langle \hat{n}_{\text{in}} \rangle \ , \tag{1}$$

where $\langle \hat{n}_{\text{in}} \rangle \gg 1$ is the average number of photons in the coherent-state input and G is the intensity gain of the amplifier. This result implies that for the case of a high-gain amplifier and a coherent-state input, the signal-to-noise ratio of the output is lower by at least a factor of 2 than that of the input. On the other hand, the shot-noise limit for a coherent state with the same average photon number as the output field is given by

$$\langle (\Delta \hat{n}_{\text{out}})^2 \rangle_{\text{SNL}} = G\langle \hat{n}_{\text{in}} \rangle \ . \tag{2}$$

The amplifier-noise limit and the shot-noise limit are shown in Fig. 1 as functions of the amplifier gain. The output of a high-gain quantum-noise-limited amplifier is seen to be significantly noisier than a coherent state with equal intensity.

The goal of squeezing experiments is to produce a beam of light for which the fluctuations in one of the field variables are below the shot-noise limit. One way to generate squeezed states is to use a parametric interaction such as four-wave mixing to first produce two beams with correlated fluctuations. The two beams are said to form twin beams if the fluctuations in the intensity difference of the beams are below the shot-noise limit [4,5]. The intensity fluctuations of one of the twin beams can be stabilized to below the shot-noise limit by adjusting its transmission through an electro-optic modulator according to the instantaneous value of the intensity of the other beam [6,7]. For such an amplitude-squeezed beam of light, the photon-number fluctuations are below the shot-noise limit shown in Fig. 1. Alternatively, the two beams can be combined with a beam splitter to generate a beam that displays less noise than a coherent state for one of the field quadrature components [8–11].

The same nonlinear interactions that are used to generate squeezed states of light can also, in principle, be used to construct optical amplifiers that improve the signal-to-noise ratio associated with the quantum-mechanical fluctuations of the field [3,12]. An amplifier that produces an output in a squeezed state has been realized experimentally in the pulsed regime [5]. In the continuous-wave regime, however, successful generation of squeezed light usually requires that a cavity be utilized to generate the two correlated beams by growth from noise [8]. Since such schemes use no external input they are not directly applicable for amplification. The minimum noise level of currently available continuous-wave optical amplifiers is thus determined by the amplifier-noise limit of Fig. 1. Therefore, in addition to generating squeezed states of light, it is of great interest to construct high-gain optical amplifiers for which the noise level of the output falls in the regime between the amplifier-noise limit and the shot-noise limit. Such amplifiers, while producing an output with more noise

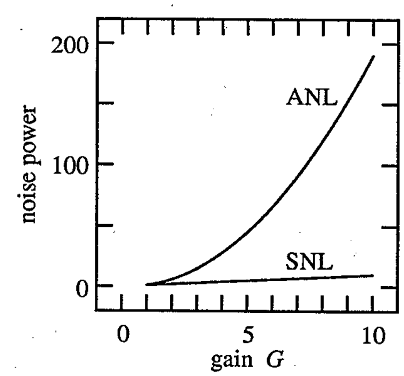

FIG. 1. The shot-noise limit (SNL) and the amplifier-noise limit (ANL) for the output of a quantum-noise-limited amplifier with gain G. The noise limits are normalized to the average number of input photons.

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than a coherent state, would offer significant improvement in the noise properties of the amplified light compared to quantum-noise-limited amplifiers.

This paper presents the results of an experimental study of a forward four-wave-mixing scheme [9] utilizing atomic potassium vapor as the nonlinear medium to amplify a continuous-wave signal beam in such a way that the noise level in the output of the amplifier is below the quantum-mechanical amplifier-noise limit. In this scheme, the four-wave-mixing process generates two output waves (amplified signal wave and generated idler wave) whose intensity fluctuations are correlated. Hence, by measuring the fluctuations of the output in the idler beam, one can determine at least part of the fluctuations of the signal beam. This correlated part of the fluctuations can then in principle be removed from the signal beam to produce a beam with reduced fluctuations [6,7]. In typical experimental situations, including the present experiment, it is more convenient to measure the temporal fluctuations rather than quantum-mechanical ensemble fluctuations of the high-intensity output beams. In such cases, in which the temporal fluctuations of the beams are of interest, the noise reduction can be accomplished by passing the amplified signal beam through an electro-optic modulator whose transmission is controlled in time according to the value of the intensity of the idler beam [6,7]. For technical reasons, we have been unable to implement such a feed-forward noise-cancellation scheme. However, we report experimental results that show that it is possible to remove the correlated part of the noise from the electronic signal after detection and we use these results to show that a properly implemented feed-forward scheme could be used to overcome the standard quantum-mechanical amplifier-noise limit.

It is important to note that the possibility of amplifying the signal beam in such a way that the standard amplifier-noise limit is overcome occurs only in a very limited quantum-mechanical sense. From a more applications oriented point of view, the proposed feed-forward scheme acts as a filter that reduces the effective noise bandwidth of the amplifier associated with the quantum-mechanical fluctuations of the output beam. The relation between these two points of view of the amplifier is elucidated by a detailed discussion of Sec. II of this paper. The experimental setup and the analysis of experimental results for the case of the potassium forward four-wave-mixing amplifier are presented in Secs. III and IV, respectively.

II. QUANTUM-MECHANICAL AND TEMPORAL FLUCTUATIONS

The fluctuation properties of high-intensity beams are conveniently investigated by the technique of spectral analysis, in which the temporal fluctuations of the beams are spectrally resolved. To properly interpret the quantum-mechanical implications of such experiments, it is necessary to distinguish between the quantum-mechanical ensemble fluctuations of a beam and the temporal fluctuations of the beam as measured by spectral analysis. The variances of photon-number fluctuations, such as the ones given by Eqs. (1) and (2), correspond to quantum-mechanical ensemble fluctuations. On the other hand, the use of a spectrum analyzer in quantum-noise measurements corresponds to determining the quantum-mechanical fluctuations of the field at a sideband frequency that is displaced by the spectrum-analyzer frequency from the carrier frequency of the field. However, it can be shown that in the case in which the bandwidth of the interaction that is used to generate the field is sufficiently larger than the spectrum-analyzer frequency, the fluctuations measured at the sideband frequency are equivalent to the quantum-mechanical ensemble fluctuations provided that both quantities are normalized to respective shot-noise levels [13]. Hence, the quantum-mechanical fluctuations of a beam can be sampled at any spectrum-analyzer frequency that is well within the bandwidth of the interaction that was used to generate the beam. Also, for such measurements the ratio of the dc power of the beam to the noise power measured within the resolution bandwidth of the spectrum analyzer is directly proportional to the signal-to-noise ratio associated with the quantum-mechanical fluctuations of the field. With regard to the forward four-wave-mixing experiment of the present paper, a sufficient amount of correlation between the temporal fluctuations of the two output beams makes it possible to construct a feedforward scheme to suppress the fluctuations in one of the beams below the level corresponding to a quantum-noise-limited amplifier. The signal-to-noise ratio of the output of a quantum-noise-limited amplifier is lower by a factor of 2 compared to that of the input. Hence, the signal-to-noise ratio of the beam is improved [12] in the process of amplification whenever the fluctuations of the amplified output are below the standard amplifier-noise limit by a factor higher than 2.

It should be noted that this possibility of overcoming the standard quantum-mechanical amplifier-noise limit occurs only in a strict quantum-mechanical sense, and that it can be interpreted in a very different way from a more applications-oriented point of view. To understand the technical implications of such an amplifier, it is instructive to consider a case in which the input signal beam carries information as temporal modulation at some frequency. It is important to note that the proposed feed-forward scheme does not allow one to reduce the temporal fluctuations of the amplified beam at the modulation frequency of the signal since the temporal modulation representing the signal would be reduced at the same proportion as the part representing temporal fluctuations. However, the feed-forward scheme allows one to reduce the effective noise bandwidth of the amplifier, i.e., to reduce the amount of fluctuations that occurs at frequencies other than that of the signal. In this respect, the correlated amplifier acts as a filter, and hence it should be no surprise that it can be used to improve the signal-to-noise ratio of the amplified output. This feature of the amplifier could be useful in all-optical applications, in which the amplified signal beam cannot be directly detected and filtered electronically.

III. EXPERIMENTAL SETUP

The experimental setup to study the noise properties of the potassium forward four-wave-mixing amplifier is shown in Fig. 2. A noncoplanar arrangement of the two pump beams (with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$) and the signal and idler beams (with wave vectors $\mathbf{k}_s$ and $\mathbf{k}_i$, respectively) is used to provide phase matching [9,14]. All beams are at the same frequency, and they are derived from a single-mode continuous-wave dye laser that is tuned close to the $4\,^2S_{1/2} \rightarrow 4\,^2P_{1/2}$ transition of potassium, which occurs at the wavelength of 767 nm. All beams intersect within a 0.5-cm-long potassium vapor cell. The beams are weakly focused with a 700-mm-focal-length lens placed ~ 90 cm before the cell. The intensities of the two pump beams are balanced to within 10%, and they are of the order of 100–150 W/cm^2 at the cell. The intensity of the input signal beam is at least a factor of 100 lower than the intensities of the pump beams. The crossing angle between the two pump beams as well as that between the signal and idler beams is $\sim 3°$.

The transmitted signal and the generated idler beams are detected with two fast photodiodes. To collect all the light in the beams, it is necessary to use a 75-mm-focal-length lens in each beam to focus the light onto the respective detector. To determine the gain of the nonlinear optical amplifier, two lock-in amplifiers are used to record simultaneously the average output power in each beam as a function of the laser detuning from the potassium linecenter. In the second set of measurements (Fig. 3), the ac components of the outputs from the two detectors are amplified and then subtracted with a 180° power combiner. The output from the power combiner is sent to a spectrum analyzer to measure the fluctuations corresponding to the intensity difference of the two beams as a function of laser detuning from resonance. To determine the noise level under identical conditions in the signal (idler) beams alone, the detector for the idler (signal) beam is blocked. All measurements are made at the spectrum-analyzer center frequency of 100 MHz to reduce the effects of the two-beam-coupling excess-noise mechanism described in Ref. [15]. After each set of noise measurements, the gain measurement is repeated to make sure that the frequency of the laser has not drifted during the measurements.

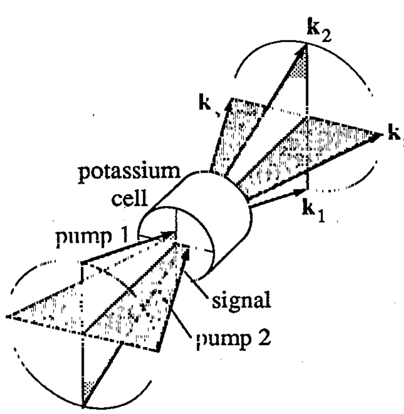

FIG. 2. The geometry of the forward four-wave-mixing interaction in a potassium vapor cell. A noncoplanar arrangement of the two pump beams (with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$) and the signal and idler beams (with wave vectors $\mathbf{k}_s$ and $\mathbf{k}_i$, respectively) is used to provide phase matching.

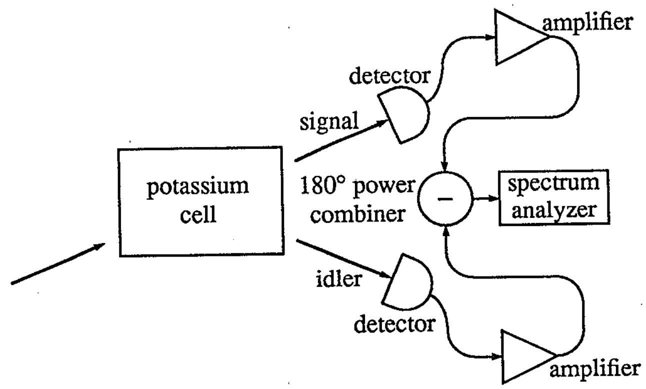

FIG. 3. The detection system for the noise measurements of the signal and idler beams. The amplified signal and the generated idler beams are detected with fast photodiodes. The resulting photocurrents are amplified, subtracted, and spectrally analyzed. To determine the noise level in the signal (idler) beam alone, the detector for the idler (signal) beam is blocked.

IV. EXPERIMENTAL RESULTS

The gain of the forward four-wave-mixing amplifier is varied most conveniently by adjusting the temperature of the potassium vapor cell. The largest gain is measured at the cell temperature of $\sim 220\,°C$. In this case, the maximum gain from the input signal beam into the output signal beam is ~ 15, and the gain from the input signal beam into the output idler beam is ~ 13. However, the correlation between the intensity fluctuations of the signal and idler beams for this case is barely sufficient that the noise level of the combined output is below the amplifier-noise limit. The noise behavior of the amplifier at this temperature (and higher) of the vapor cell is found to be limited by strong self-focusing and self-defocusing effects, which make it difficult to properly separate the signal and idler beams from the two pump beams after propagation through the cell.

The best noise performance of the amplifier is obtained at the cell temperature of $\sim 200\,°C$. The gains measured at this temperature for the two arms of the amplifier are shown in Fig. 4(a) as functions of the laser detuning from the potassium line center. In this case, the maximum gain for the signal arm is ~ 6.2, and the maximum gain for the idler arm is ~ 5.4. We believe that the asymmetry between the results obtained on the red and blue sides of the line center, which is evident in Fig. 4(a), is due to self-defocusing and self-focusing effects. In Fig. 4(b), the measured noise power in the signal arm is shown as a function of the laser detuning. The measured noise level is compared to the shot-noise limit and amplifier-noise limit of the output beam, which are calculated using Eqs. (1) and (2) and the measured gain spectrum shown in Fig. 4(a). A similar plot for the idler arm is shown in Fig. 4(c). The noise levels are normalized to the off-resonance shot-noise limit of the signal arm. It is important to note that for the detuning corresponding to the maximum gain, both of the outputs beams are noisier than that of a quantum-noise-limited amplifier by a factor of ~ 2.5. This excessive amount of noise is most likely due to

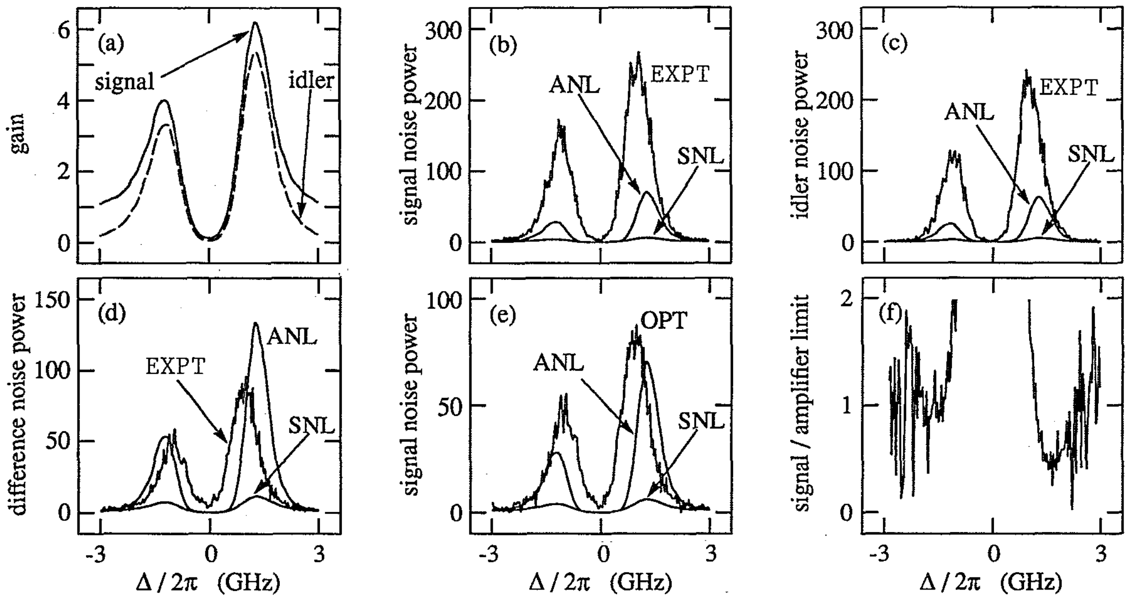

FIG. 4. (a) Gain of signal and idler arms; (b) measured noise power (EXPT), amplifier-noise limit (ANL), and shot-noise limit (SNL) for signal arm; (c) same as (b), but for idler arm; (d) same as (b), but for the difference between the two arms; (e) optimized noise power of signal arm (OPT) that can be achieved with a feed-forward scheme compared to ANL and SNL of case (b); (f) ratio of optimized noise power to the amplifier-noise limit for the signal arm. All plots are as functions of laser detuning Δ from the line center. Spectrum-analyzer frequency is $\Omega=100$ MHz. Noise levels are normalized to the off-resonance shot-noise limit of the input signal beam.

atomic fluctuations [16]. The noise plots are also seen to peak closer to the line center than the gain plots (and the noise-limit plots, which are determined from the gain plots). We believe that this effect is due to the fact that the noise of the signal and idler beams arising from atomic fluctuations is enhanced close to the line center even though the gain experienced by the beams is degraded by increased absorption. In Fig. 4(d), the noise level of the intensity difference of the two output beams is compared to the shot-noise limit and the amplifier-noise limit that have been determined theoretically for the fictional case of two uncorrelated and quantum-noise-limited amplifiers whose gains are assumed to be equal to the gains of the signal and idler arms of the potassium four-wave-mixing amplifier. The measured noise level is seen to be below that of the amplifier-noise limit for a range of detunings around ~1.5 GHz to the blue side of exact resonance. This result implies that for this range of detunings, there is a significant amount of correlation between the fluctuations of the two outputs beams of the amplifier.

We next determine how well an electro-optic feed-forward scheme [6,7] could stabilize the intensity fluctuations of the signal beam when the idler beam is used as a reference. The theoretical capability of such a scheme to suppress the fluctuations at frequency Ω can be determined by measuring the power spectra of the intensity fluctuations of the signal $[S_s(\Omega)]$ and idler $[S_i(\Omega)]$ beams, and the power spectrum corresponding to the fluctuations of the intensity difference of the two beams $[S_{s\text{-}i}(\Omega)]$. The optimum value of the power spectrum of the corrected signal beam is given by [7]

$$S_s^{\rm opt}(\Omega)=\frac{S_s(\Omega)S_i(\Omega)-|C_{s,i}(\Omega)|^2}{S_i(\Omega)}\ , \quad (3)$$

where $C_{s,i}(\Omega)$ is a correlation function that characterizes the correlations between the fluctuations of the signal and idler beams. The case where the correlation function is purely real corresponds to the worst-case situation for correcting the intensity fluctuations of the signal beam. For this worst-case situation, the correlation function can be shown to be given by

$$C_{s,i}(\Omega)=\tfrac{1}{2}[S_s(\Omega)+S_i(\Omega)-S_{s\text{-}i}(\Omega)]\ . \quad (4)$$

The experimentally measured spectra shown in Figs. 4(b)–4(d) are used to calculate the optimized noise level of the signal beam using Eqs. (3) and (4). The result is illustrated in Fig. 4(e), where it is compared to the shot-noise level and amplifier-noise level of Fig. 4(b), which correspond to a single quantum-noise-limited amplifier. The optimized noise spectrum is also shown normalized to the amplifier-noise limit in Fig. 4(f). The correlation between the intensity fluctuations of the signal and idler beams is seen to be sufficient that the intensity fluctuations of the signal beam alone could be stabilized to a level that is ~60% below the usual amplifier-noise limit for a small range of detunings around ~1.5 GHz to the blue side of the potassium linecenter. By the discussion of Sec. II, the present results imply that the signal-to-noise ratio associated with the quantum-mechanical fluctuations of the beam could be improved in the process of amplification.

V. CONCLUSIONS

The results of this investigation show that forward four-wave mixing utilizing an atomic vapor as the nonlinear medium can be used to construct a low-noise continuous-wave optical amplifier with high gain and low noise. From a strictly quantum-mechanical point of view, the correlations between the intensity fluctuations of the two output beams of the amplifier make it possible to construct an amplifier with less noise than a quantum-noise-limited amplifier. From a more applications-oriented point of view, this result can be interpreted as a filtering mechanism in which the correlations between the two outputs of the amplifier are used to reduce the effective noise bandwidth of the amplifier.

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