Exploring Energy-Time Entanglement Using Geometric Phase

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Background: Geometric Phase

- **Geometric Phase (Berry’s Phase)**

- **Pancharatnam’s Phase**

- **Connection**
  M.V. Berry, J. Mod. Opt. 34, 1401 (1987)
  Ramaseshan and Nityananda, Curr. Sci. 55, 1225 (1986)

- **Example:**
  Tomita and Chiao, PRL 57, 937 (1986)
  Bhandari and Samuel, PRL 60, 1211 (1988)

Pancharatnam’s phase is the geometric phase in polarization optics.

Acquired by a system in an eigenstate when transported around a closed circuit by varying some parameters in its Hamiltonian.

Acquired by a light field when its state of polarization is taken through a closed circuit on the Poincare sphere.
Background: Energy-Time Entanglement

Energy conservation

\[ \omega_p = \omega_s + \omega_i \]

Energy-Time Entanglement

\[ |\psi\rangle = \int d\omega \phi(\omega) |\omega\rangle_s |\omega_p - \omega\rangle_i \]

\[ |\psi\rangle = \int dt f(t) |t\rangle_s |t\rangle_i \]

- Bell inequality for Energy and Time

State

\[ |\psi\rangle = \frac{1}{\sqrt{2}} [ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i ] \]

Coincidence count rate

\[ R_{AB} = C[1 + \cos(\phi_s + \phi_i)] \]

Violation of CHSH Bell inequality
(as entanglement witness)

Brendel et al., PRL 66, 1142 (1991)
Kwiat et al., PRA 47, R2472 (1993)
Strekalov et al., PRA 54, R1 (1996)
Barreiro et al., PRL 95, 260501 (2005)

Dynamic phase based violation

Franson, PRL 62, 2205 (1989)
Geometric phase based violation

State:

\[ | \psi \rangle = \frac{1}{\sqrt{2}} \left[ | l \rangle_s | l \rangle_i + | s \rangle_s | s \rangle_i \right] \]

Coincidence count rate:

\[ R_{AB} = C \{ 1 - \cos[k_0(x_s + x_i) - 2\beta_s - 2\beta_i] \} \]

dynamic phase  geometric phase

Jha, O'Sullivan, Chan, and Boyd, PRA 77, 021801(R) (2008)
Geometric phase based violation

State:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} [ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i ] \]

Coincidence count rate:

\[ R_{AB} = C \{ 1 + \cos[k_0(x_s - x_i)] + 2\beta_s + 2\beta_i \} \]

\[ V = 77\% \ (\geq 70.7\%) \]
\[ S = 2\sqrt{2}V \]
\[ = 2.18 \pm 0.04 \ (\geq 2.0) \]

Violation by 5 standard deviations

Jha, O’Sullivan, Chan, and Boyd, PRA 77, 021801(R) (2008)
Jha, Malik, and Boyd, PRL 101, 180405 (2008)
Conclusions

- Energy-Time Entanglement can be explored using geometric phase

- Potential benefits for quantum information science
  - Geometric phase is non-dispersive (wavelength independent)
  - Ease in introducing small phase shifts
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http://www.optics.rochester.edu/~boyd