

# Fundamentals and Applications of Slow Light and An Introduction to Quantum Imaging

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Presented at the MIT-Harvard Center for Ultracold Atoms, May 15, 2007.

# Interest in Slow Light

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Intrigue: Can (group) refractive index really be  $10^6$ ?

Fundamentals of optical physics

Optical delay lines, optical storage, optical memories

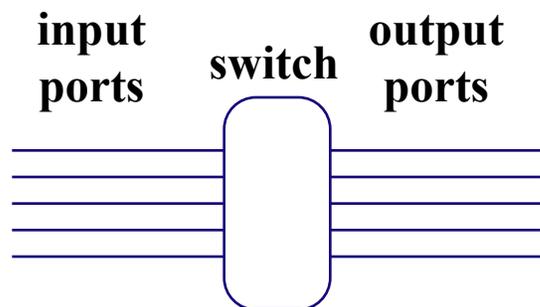
Implications for quantum information

What about fast light ( $v > c$ ) and backwards light ( $v$  negative)?

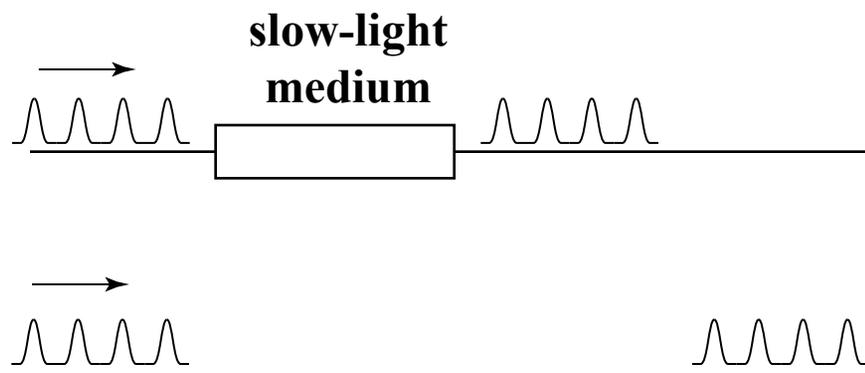
Boyd and Gauthier, "Slow and Fast Light," in Progress in Optics, 43, 2002.



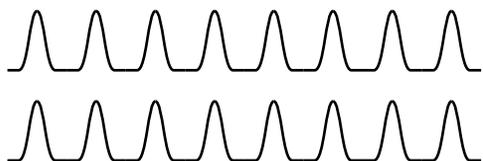
## All-Optical Switch



## Use Optical Buffering to Resolve Data-Packet Contention



**But what happens if two data packets arrive simultaneously?**



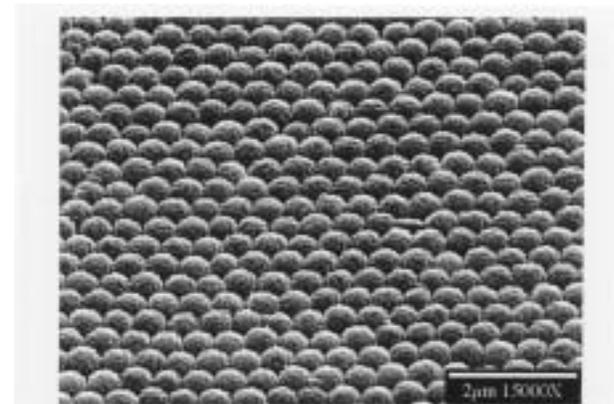
**Controllable slow light for optical buffering can dramatically increase system performance.**

# Some Approaches to Slow Light Propagation

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- Use the linear response of atomic systems  
or (better)  
use quantum coherence (e.g., electromagnetically induced transparency) to modify and control this response
- Use of artificial materials (to modify the optical properties at the macroscopic level)

E.g., photonic crystals where strong spectral variation of the refractive index occurs near the edge of the photonic bandgap



polystyrene photonic crystal

# Slow and Fast Light and Optical Resonances

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Pulses propagate at the group velocity given by

$$v_g = \frac{c}{n_g} \quad n_g = n + \omega \frac{dn}{d\omega}$$

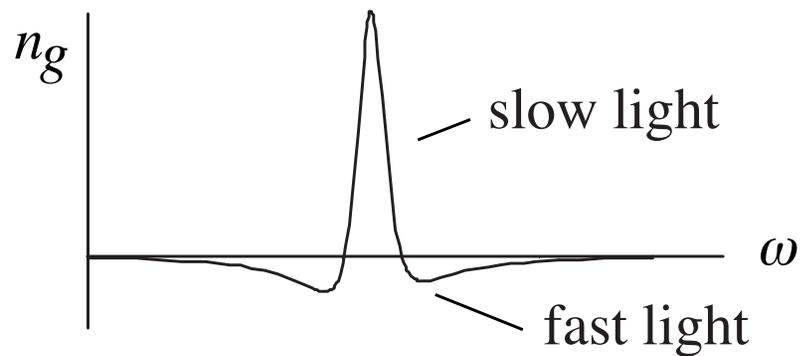
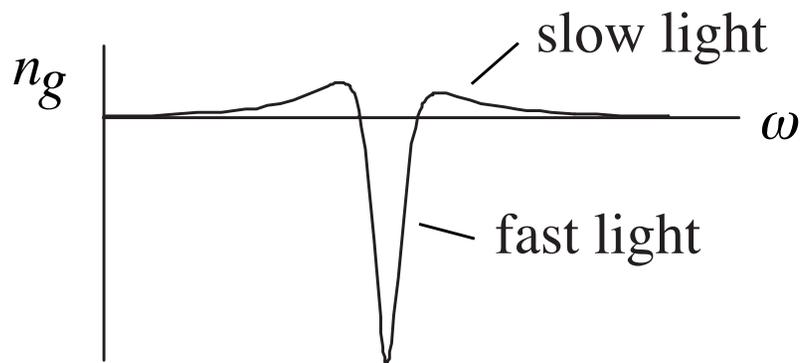
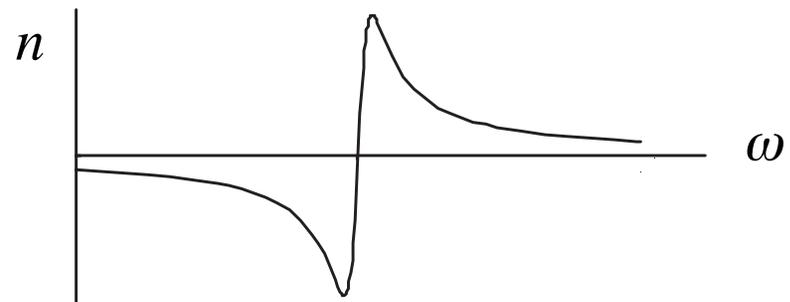
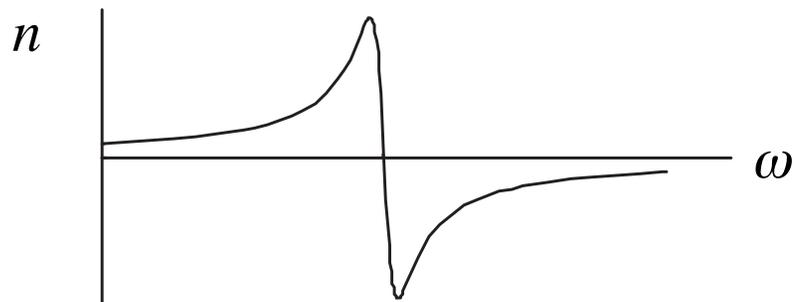
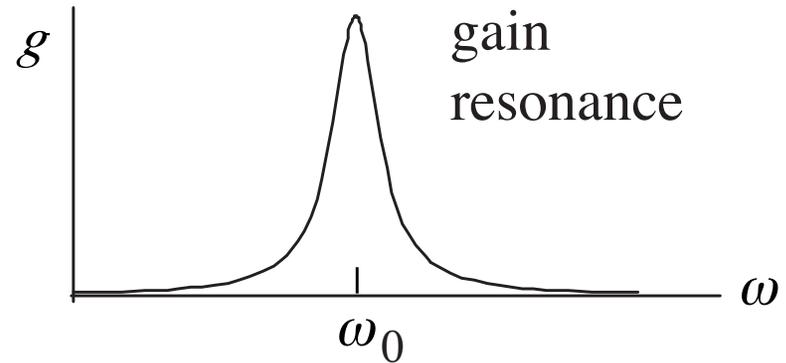
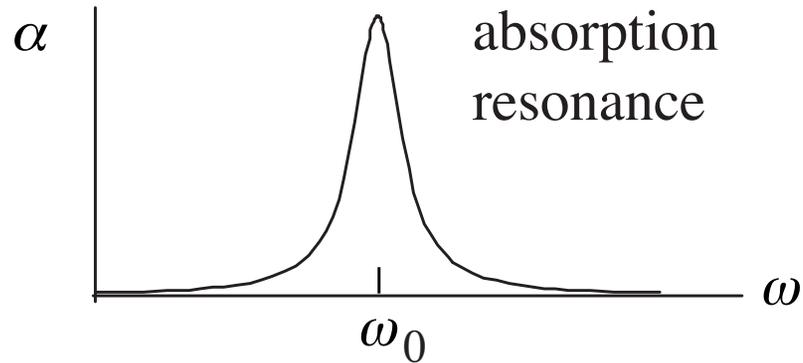
Want large dispersion to obtain extreme group velocities

Sharp spectral features produce large dispersion.

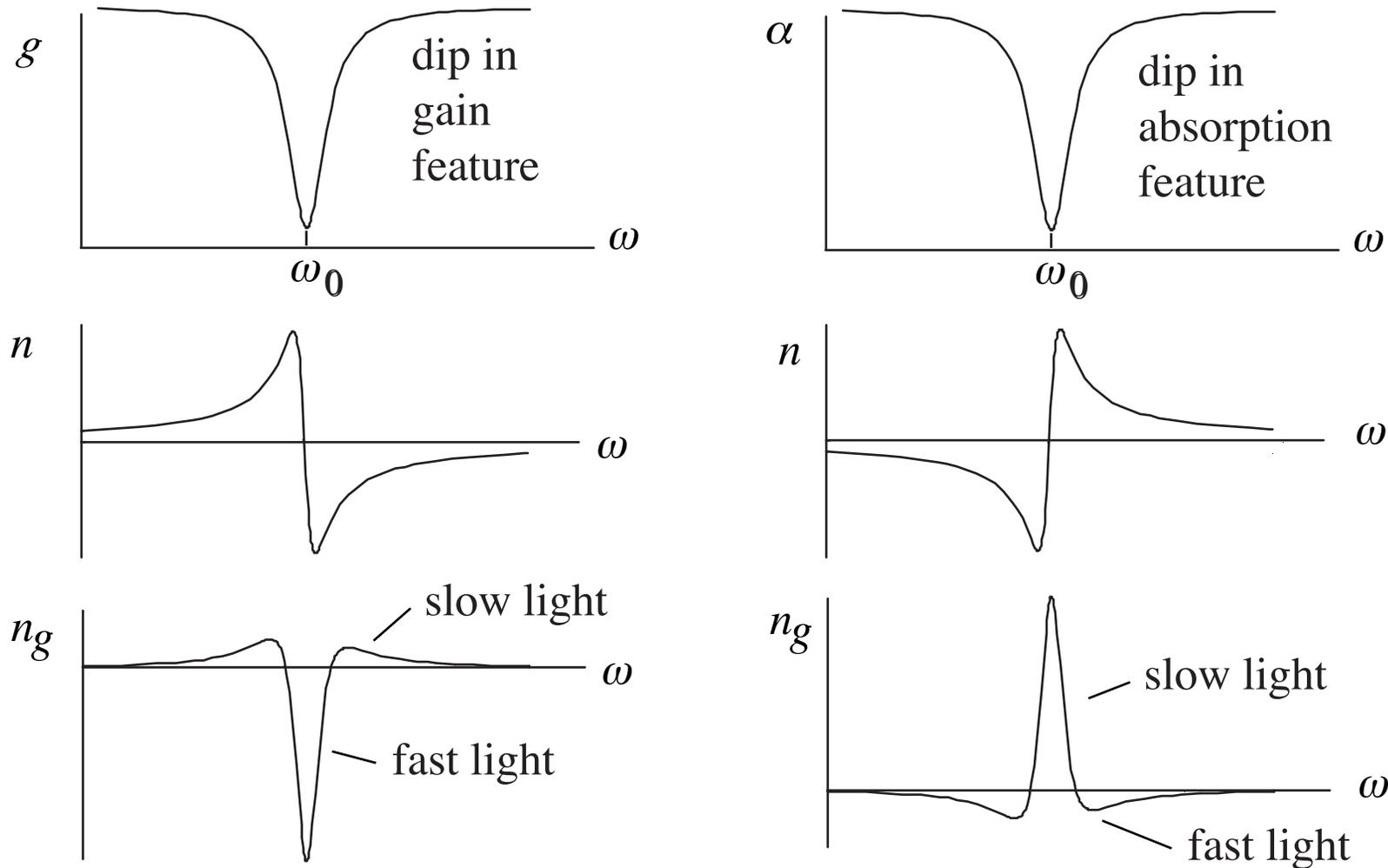
The group index can be large and positive (slow light), positive and much less than unity (fast light) or negative (backwards light).

# How to Create Slow and Fast Light I – Use Isolated Gain or Absorption Resonance

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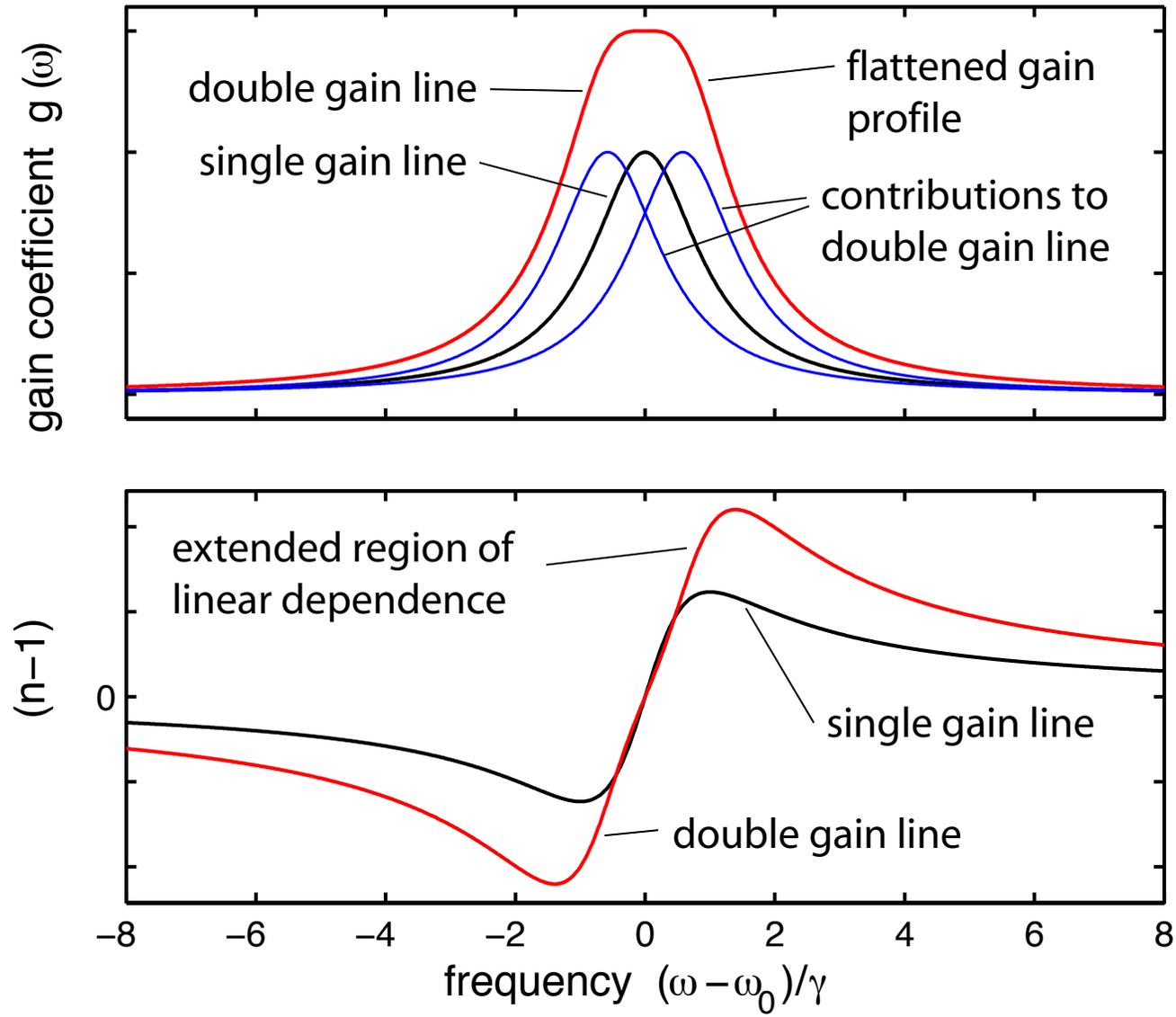


# How to Create Slow and Fast Light II – Use Dip in Gain or Absorption Feature



Narrow dips in gain and absorption lines can be created by various nonlinear optical effects, such as electromagnetically induced transparency (EIT), coherent population oscillations (CPO), and conventional saturation.

# How to Create Slow and Fast Light III – Dispersion Management



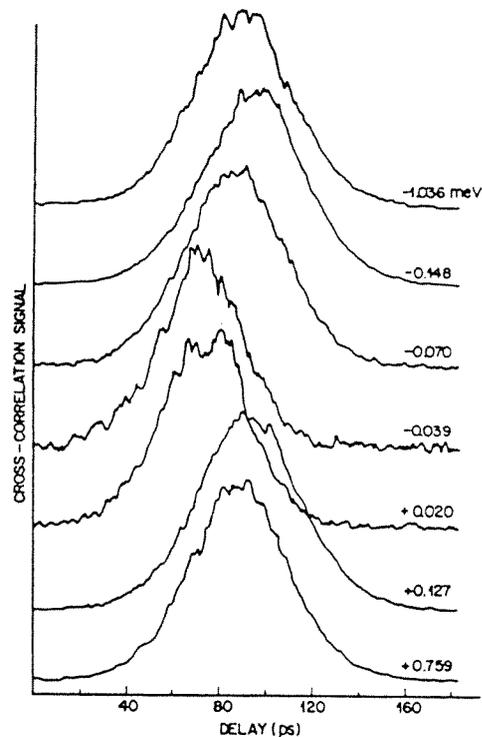
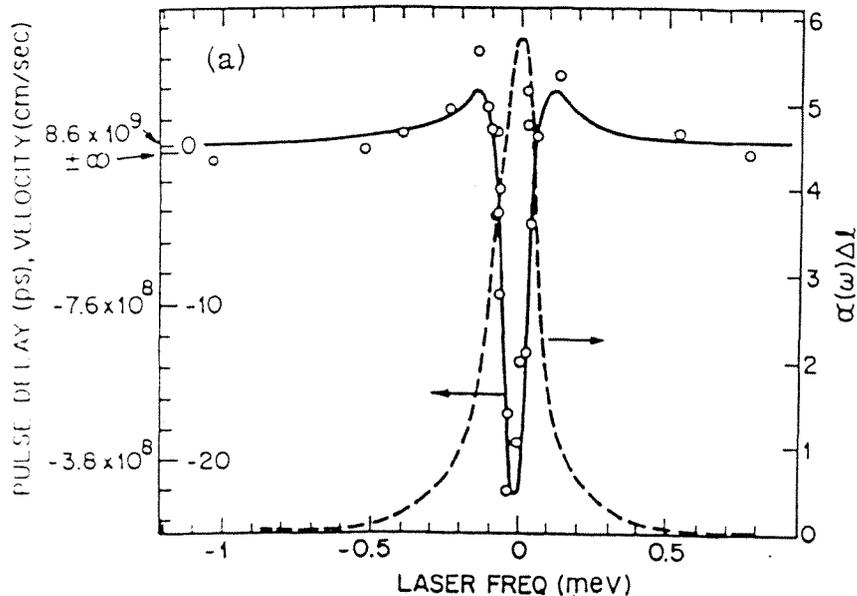
# Linear Pulse Propagation in an Absorbing Medium

S. Chu and S. Wong

*Bell Laboratories, Murray Hill, New Jersey 07974*

(Received 30 November 1981)

The pulse velocity in the linear regime in samples of GaP:N with a laser tuned to the bound A-exciton line is measured with use of a picosecond time-of-flight technique. The pulse is seen to propagate through the material with little pulse-shape distortion, and with an envelope velocity given by the group velocity even when the group velocity exceeds  $3 \times 10^{10}$  cm/sec, equals  $\pm \infty$ , or becomes negative. The results verify the predictions of Garrett and McCumber.



# Light speed reduction to 17 metres per second in an ultracold atomic gas

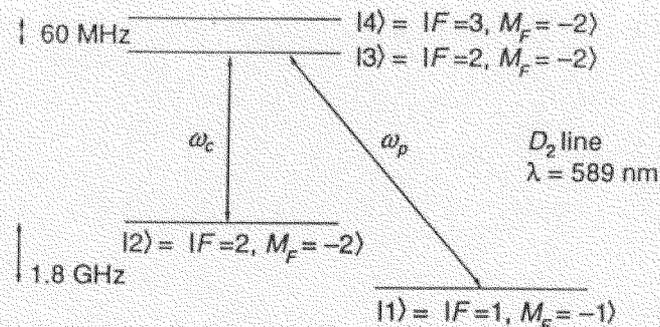
Lene Vestergaard Hau<sup>\*†</sup>, S. E. Harris<sup>‡</sup>, Zachary Dutton<sup>\*†</sup>  
& Cyrus H. Behroozi<sup>\*§</sup>

<sup>\*</sup> Rowland Institute for Science, 100 Edwin H. Land Boulevard, Cambridge,  
Massachusetts 02142, USA

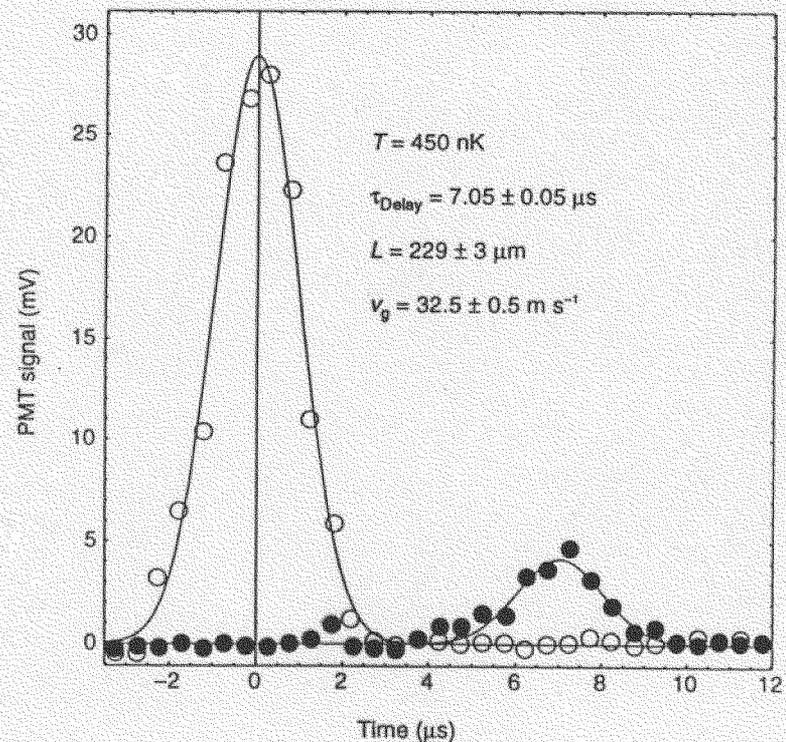
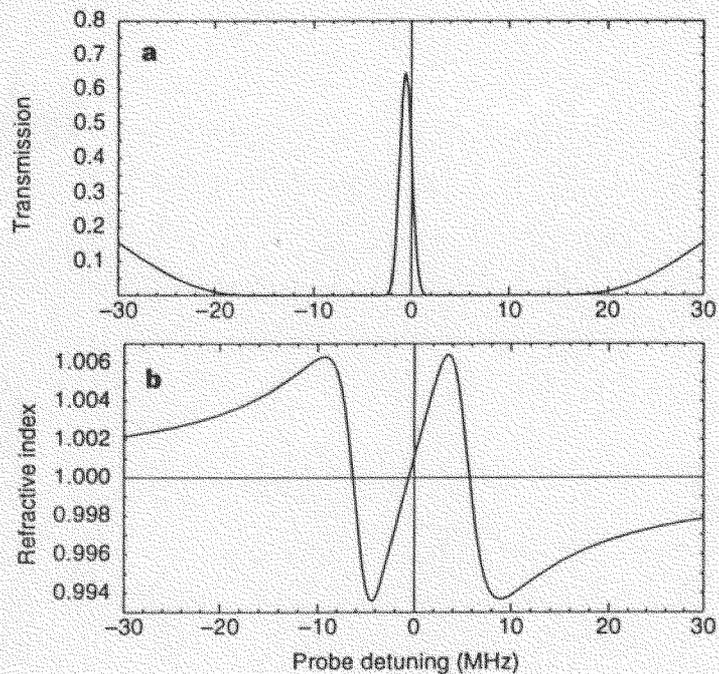
<sup>†</sup> Department of Physics, <sup>§</sup> Division of Engineering and Applied Sciences,  
Harvard University, Cambridge, Massachusetts 02138, USA

<sup>‡</sup> Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305,  
USA

Nature, 397, 594, (1999).



$$v_g = \frac{c}{n(\omega_p) + \omega_p \frac{dn}{d\omega_p}} \approx \frac{\hbar c \epsilon_0 |\Omega_c|^2}{2\omega_p |\mu_{13}|^2 N}$$



## Amplification of Light and Atoms in a Bose-Einstein Condensate

S. Inouye, R. F. Löw, S. Gupta, T. Pfau, A. Görlitz, T. L. Gustavson, D. E. Pritchard, and W. Ketterle

*Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology,  
Cambridge, Massachusetts 02139*

(Received 27 June 2000)

A Bose-Einstein condensate illuminated by a single off-resonant laser beam (“dressed condensate”) shows a high gain for matter waves and light. We have characterized the optical and atom-optical properties of the dressed condensate by injecting light or atoms, illuminating the key role of long-lived matter wave gratings produced by the condensate at rest and recoiling atoms. The narrow bandwidth for optical gain gave rise to an extremely slow group velocity of an amplified light pulse ( $\sim 1$  m/s).

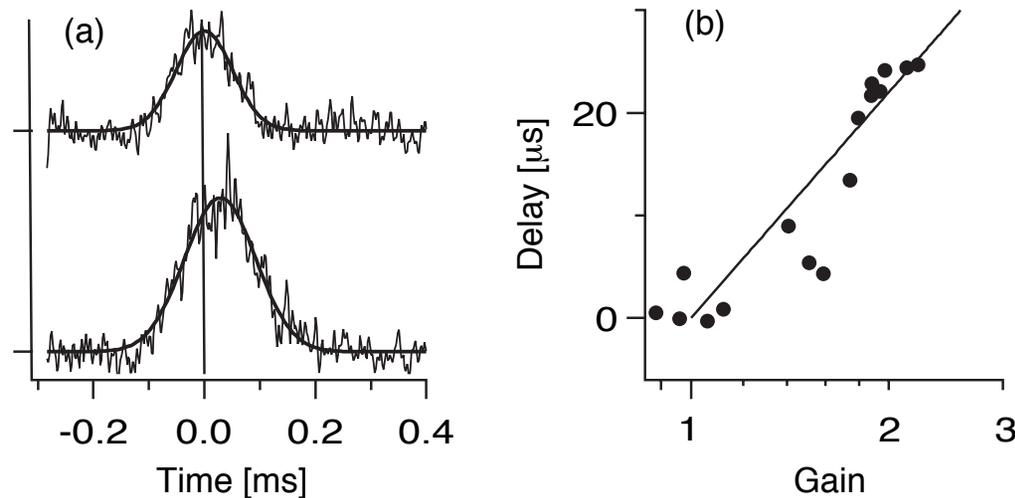


FIG. 3. Pulse delay due to light amplification. (a) About 20 ms delay was observed when a Gaussian pulse of about 140 ms width and  $0.11 \text{ mW/cm}^2$  peak intensity was sent through the dressed condensate (bottom trace). The top trace is a reference taken without the dressed condensate. Solid curves are Gaussian fits to guide the eyes. (b) The observed delay  $t_D$  was proportional to  $(\ln g)$ , where  $g$  is the observed gain.

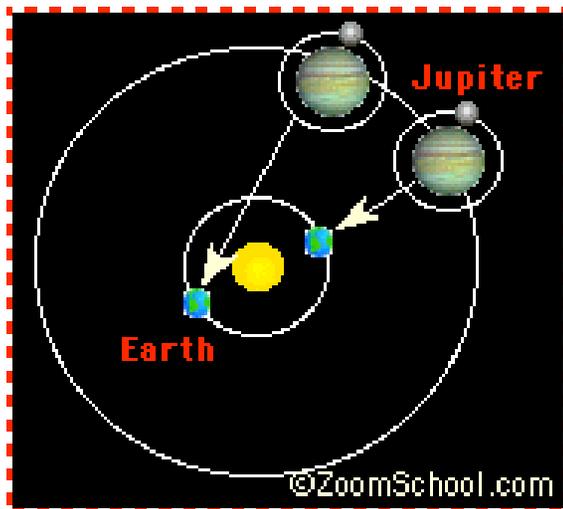
# Determination of the Velocity of Light\*

## “Astronomical” Methods

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Römer (1676) First evidence that velocity of light is finite!

Observed an apparent variation of up to 22 minutes in the orbital period of the satellite Io in its orbit about Jupiter.



Deduced that  $c = 225,000$  km/sec

(Actually, light transit time from sun to earth is just over 8 minutes, and  $c = 299,793$  km/sec)

\*See, for instance, Jenkins and White, 1976.

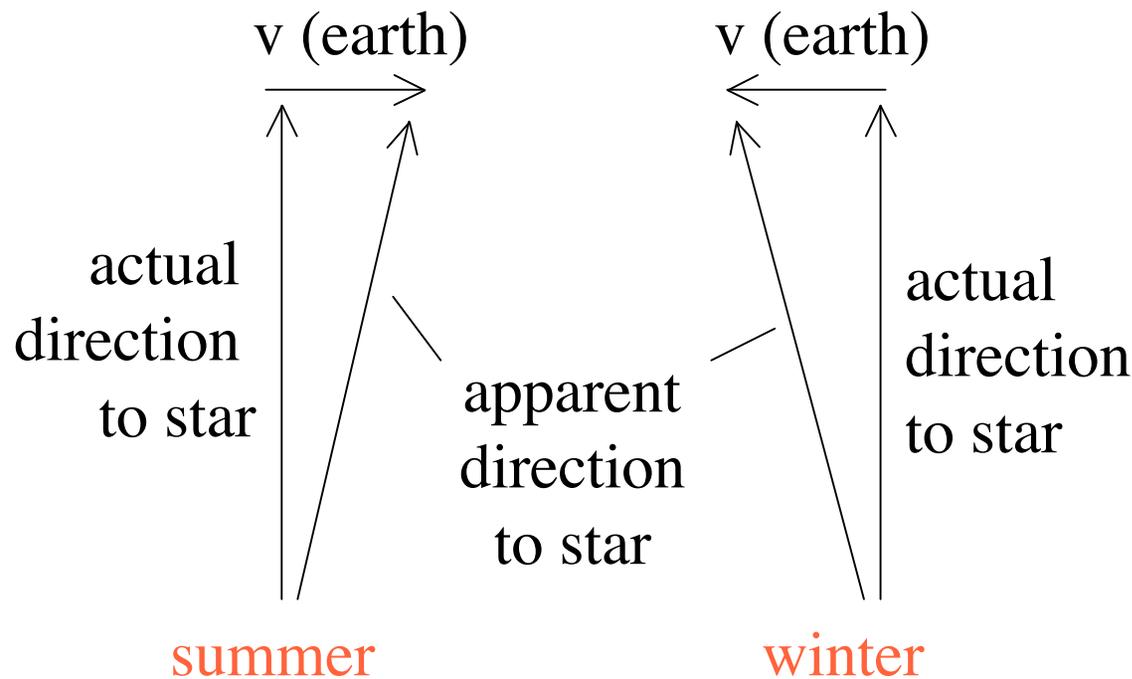
# Determination of the Velocity of Light

## Astronomical Methods

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Bradley (1727); Aberration of star light.

Confirmation of the finite velocity of light.



$$v(\text{earth}) \approx 30 \text{ km/s}$$

$$\tan \alpha = \frac{v(\text{earth})}{c}$$

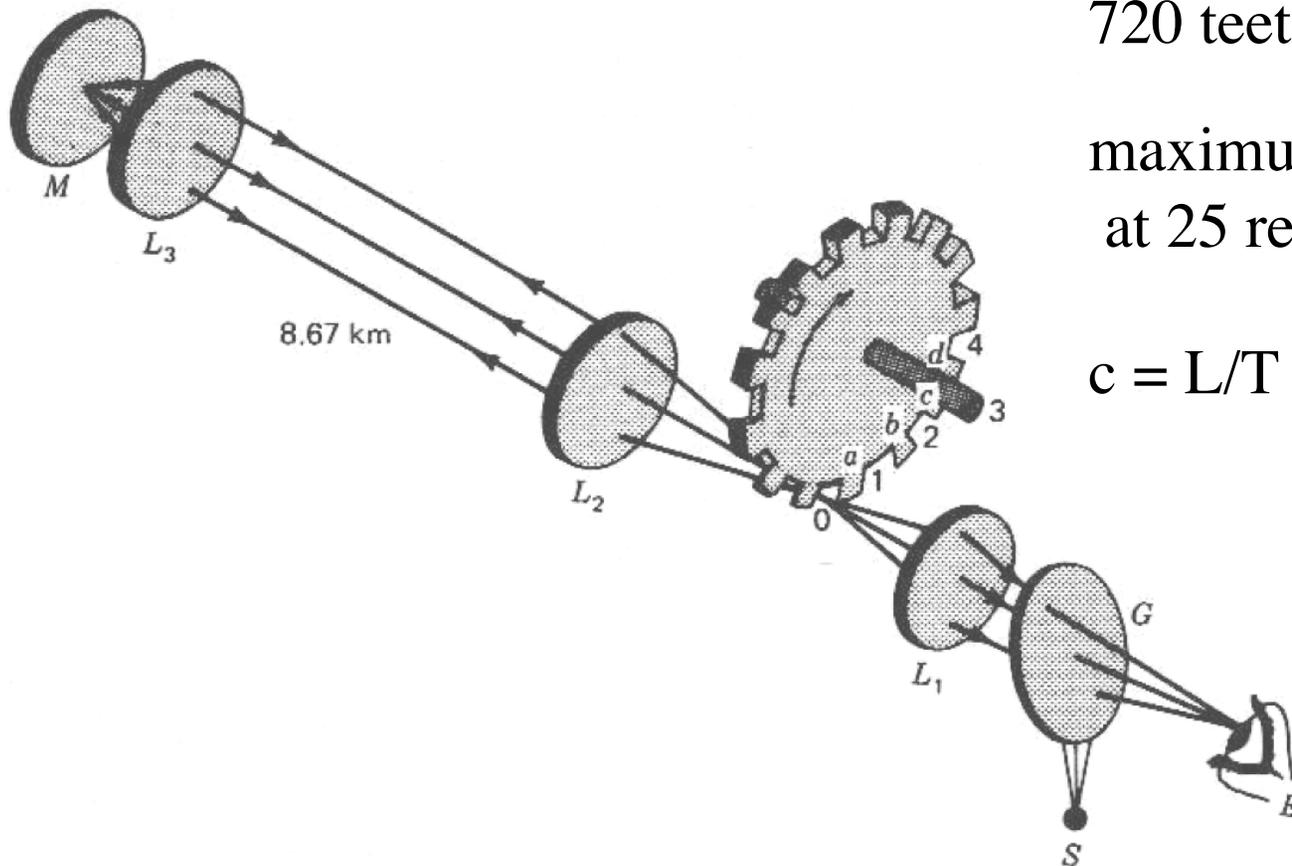
$$\alpha = 20.5 \text{ arcsec}$$

# Determination of the Velocity of Light

## Laboratory Methods

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Fizeau (1849) Time-of-flight method



720 teeth in wheel

maximum transmission  
at 25 revolutions/sec

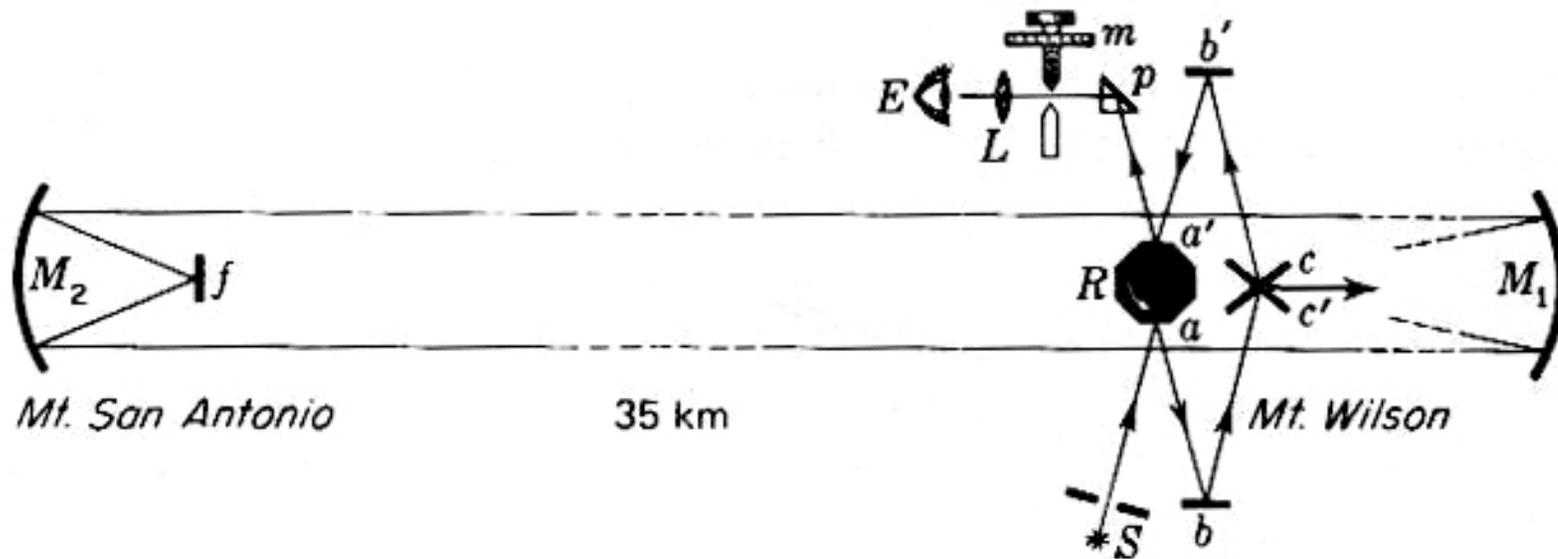
$$c = L/T = 320,000 \text{ km/s}$$

# Determination of the Velocity of Light

## Laboratory Methods

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Michelson (1926); Improved time of flight method.



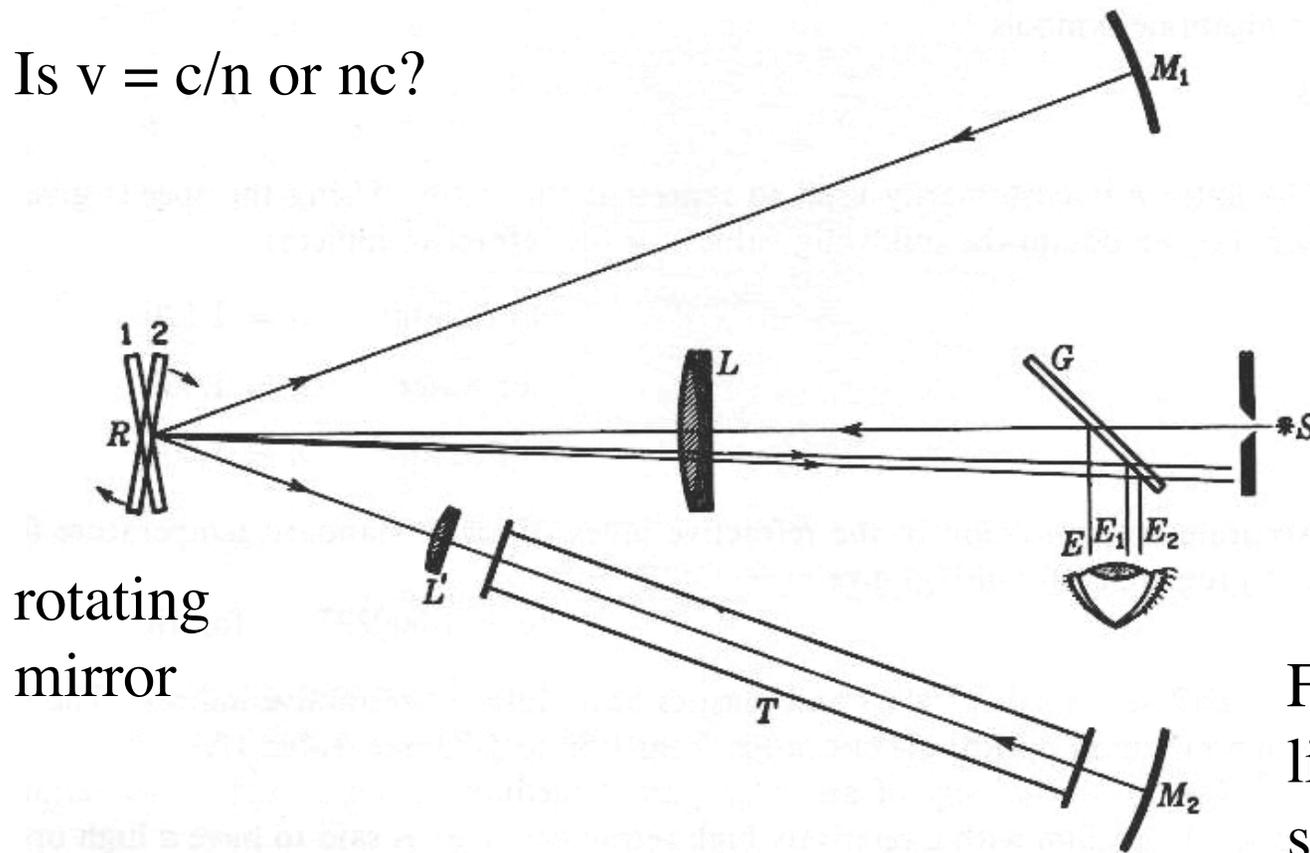
Rotating octagonal mirror

$$c = 299,296 \text{ km/s (or } 299,298 \text{ km/s)}$$

# Velocity of Light in Matter

Foucault (1850) Velocity of light in water.

Is  $v = c/n$  or  $nc$ ?



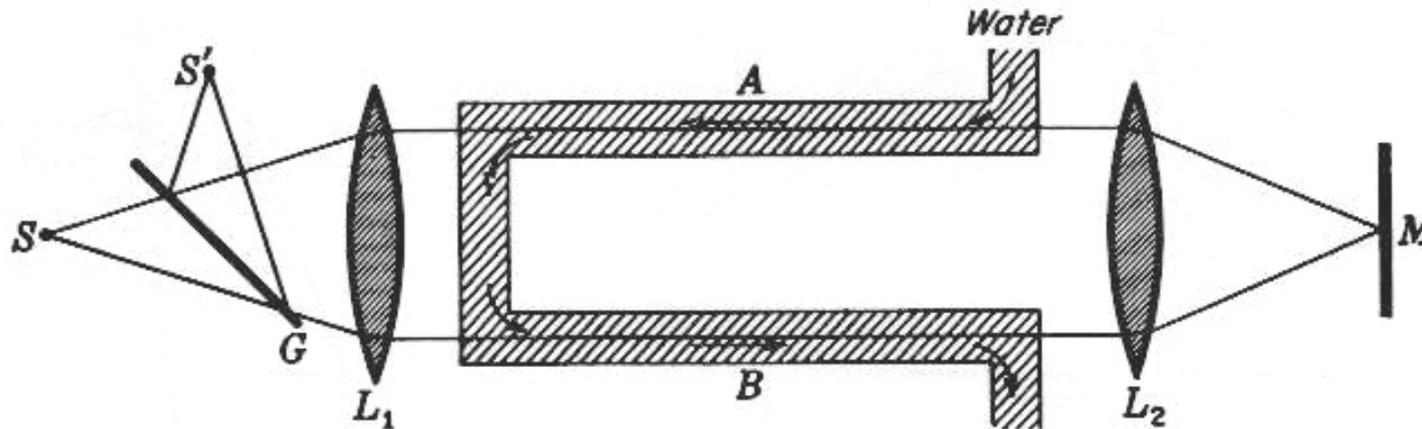
rotating  
mirror

Foucault finds that  
light travels more  
slowly in water!

# Velocity of Light in Moving Matter

Fizeau (1859); Velocity of light in flowing water.

$V = 700 \text{ cm/sec}$ ;  $L = 150 \text{ cm}$ ; displacement of 0.5 fringe.



Modern theory: relativistic addition of velocities

$$v = \frac{c/n + V}{1 + (V/c)(1/n)} \approx \frac{c}{n} + V \left( 1 - \frac{1}{n^2} \right)$$

Fresnel “drag” coefficient

## Challenge / Goal (2003)

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Slow light is a room-temperature, solid-state material.

Our solution:

Slow light *via* coherent population oscillations (CPO), a quantum coherence effect related to EIT but which is less sensitive to dephasing processes.

# Slow Light in Ruby

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Recall that  $n_g = n + \omega(dn/d\omega)$ . Need a large  $dn/d\omega$ . (How?)

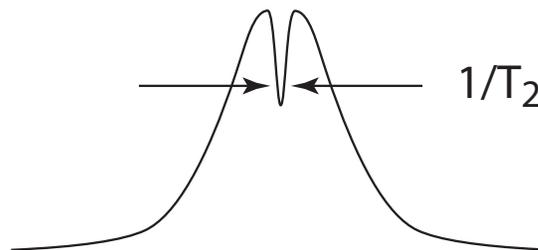
Kramers-Kronig relations:

Want a very narrow feature in absorption line.

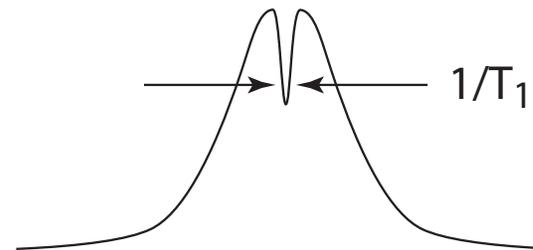
Well-known “trick” for doing so:

Make use of spectral holes due to population oscillations.

Hole-burning in a homogeneously broadened line; requires  $T_2 \ll T_1$ .



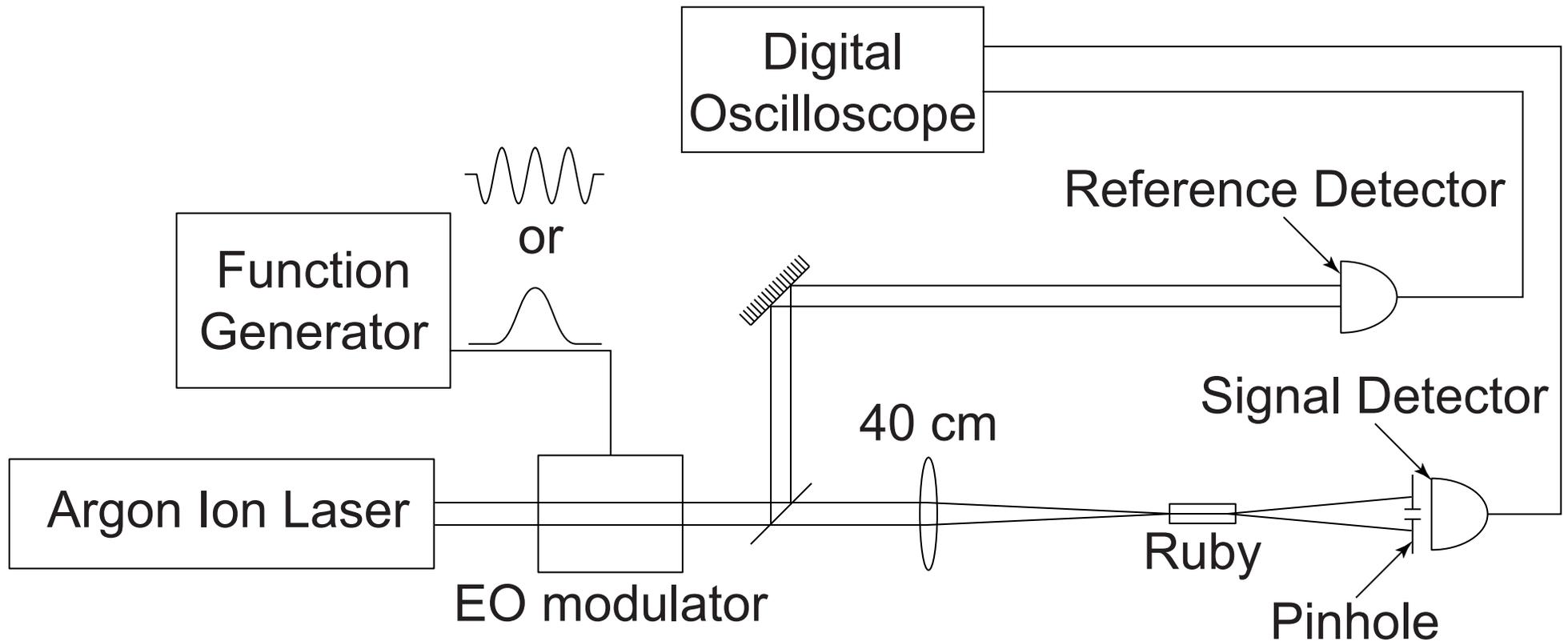
inhomogeneously  
broadened medium



homogeneously  
broadened medium  
(or inhomogeneously  
broadened)

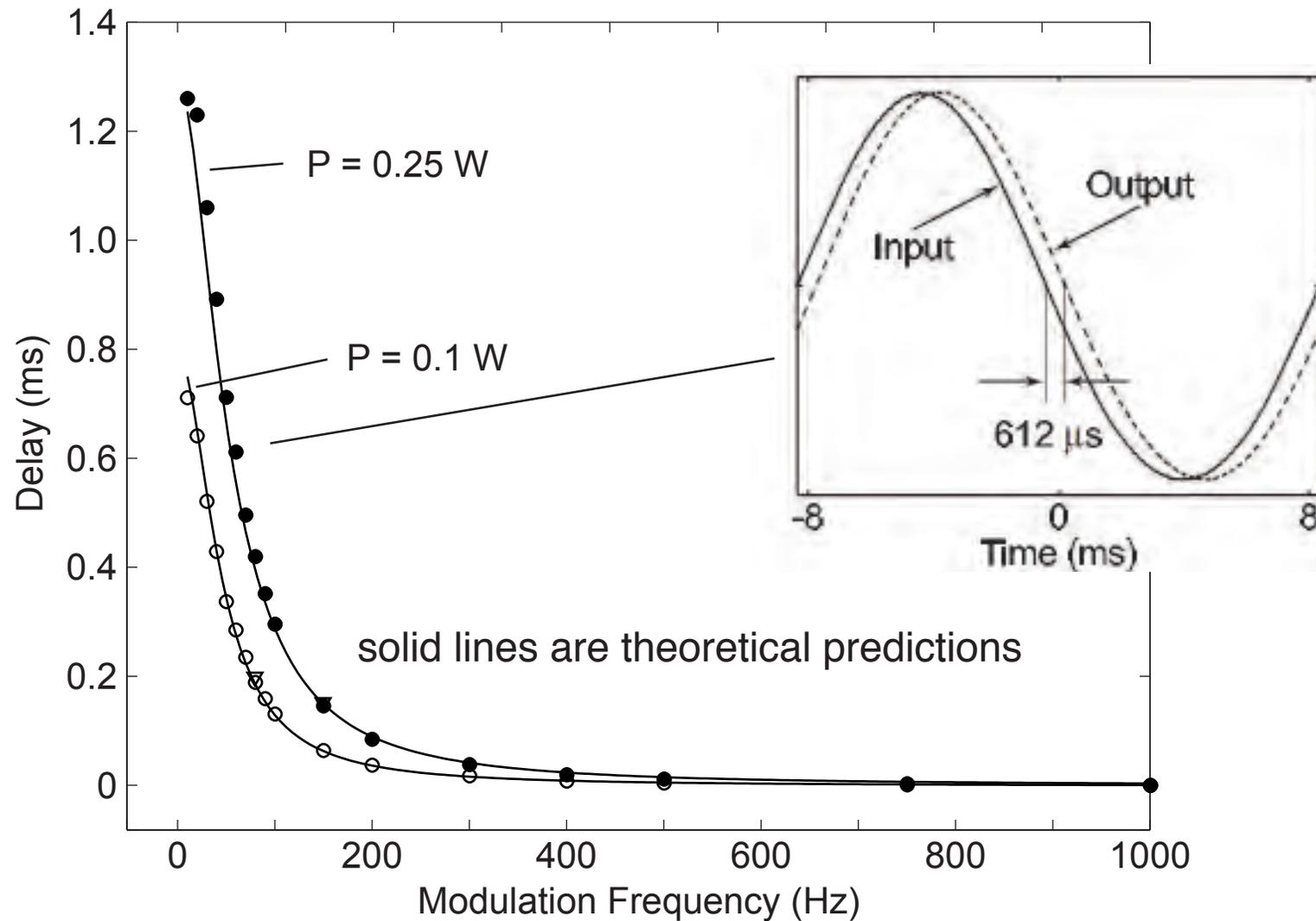
# Slow Light Experimental Setup

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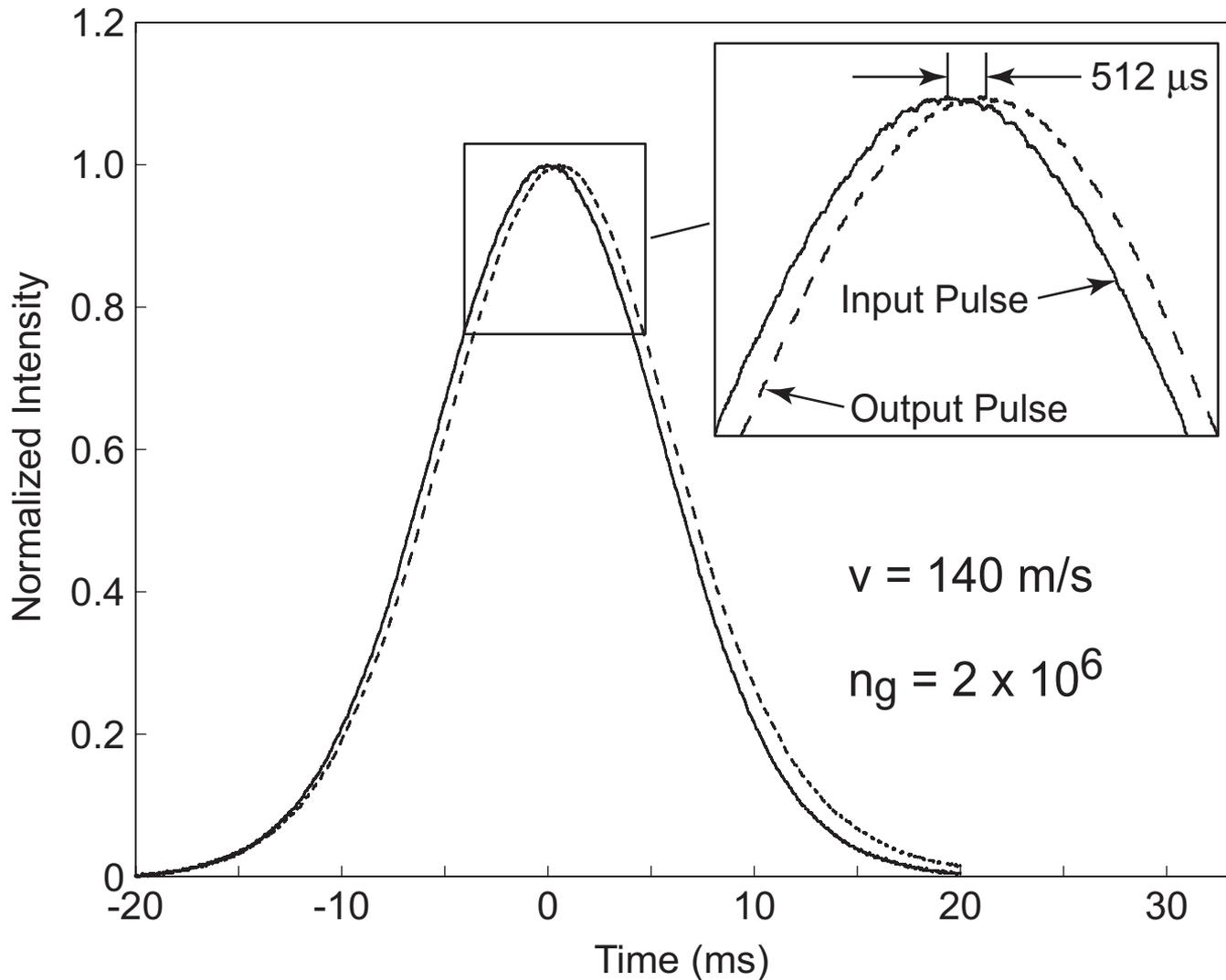
7.25-cm-long ruby laser rod (pink ruby)

# Measurement of Delay Time for Harmonic Modulation



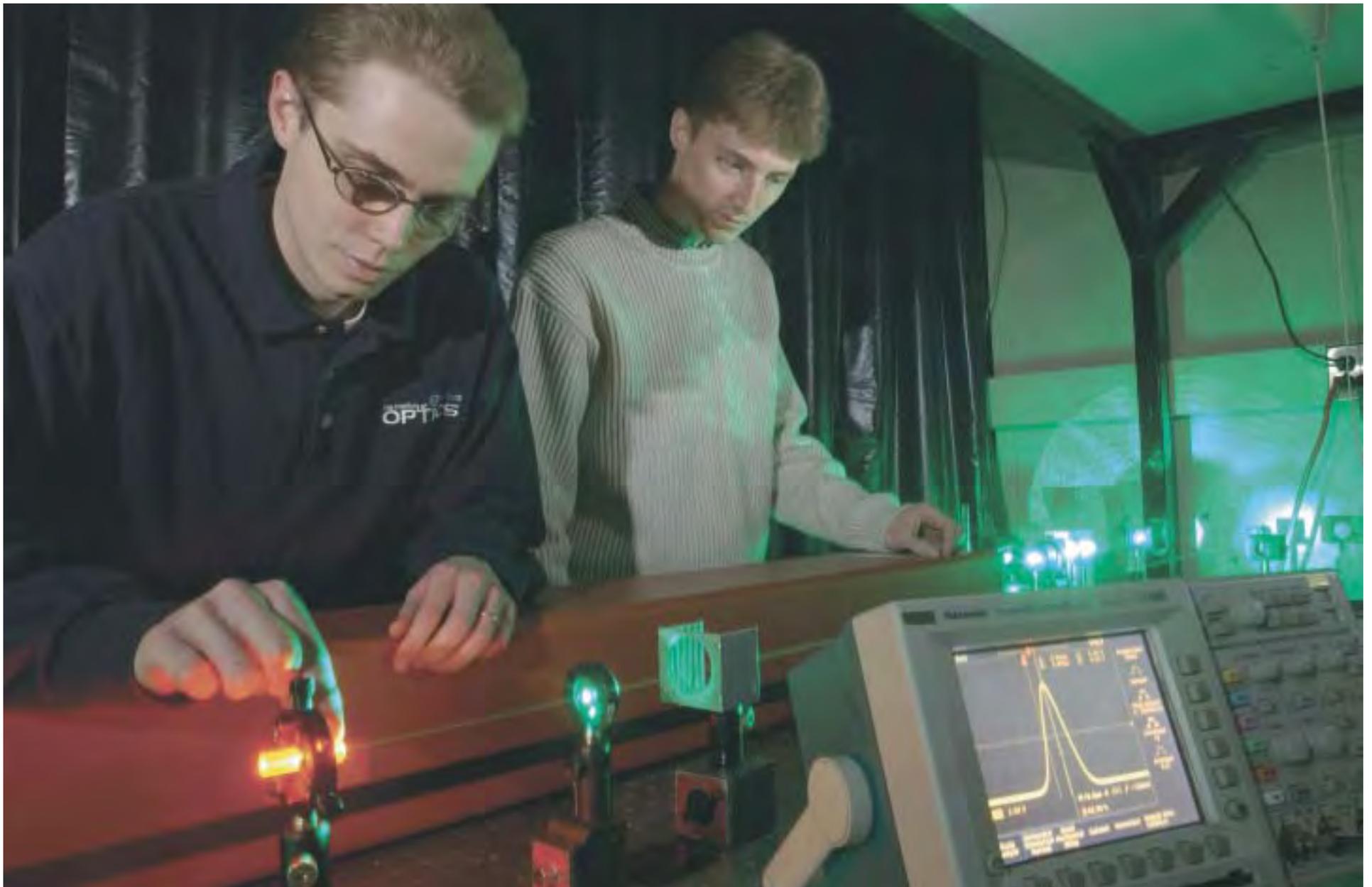
For 1.2 ms delay,  $v = 60 \text{ m/s}$  and  $n_g = 5 \times 10^6$

# Gaussian Pulse Propagation Through Ruby



No pulse distortion!

# Matt Bigelow and Nick Lepeshkin in the Lab



# **Advantages of Coherent Population Oscillations for Slow Light**

**Works in solids**

**Works at room temperature**

**Insensitive of dephasing processes**

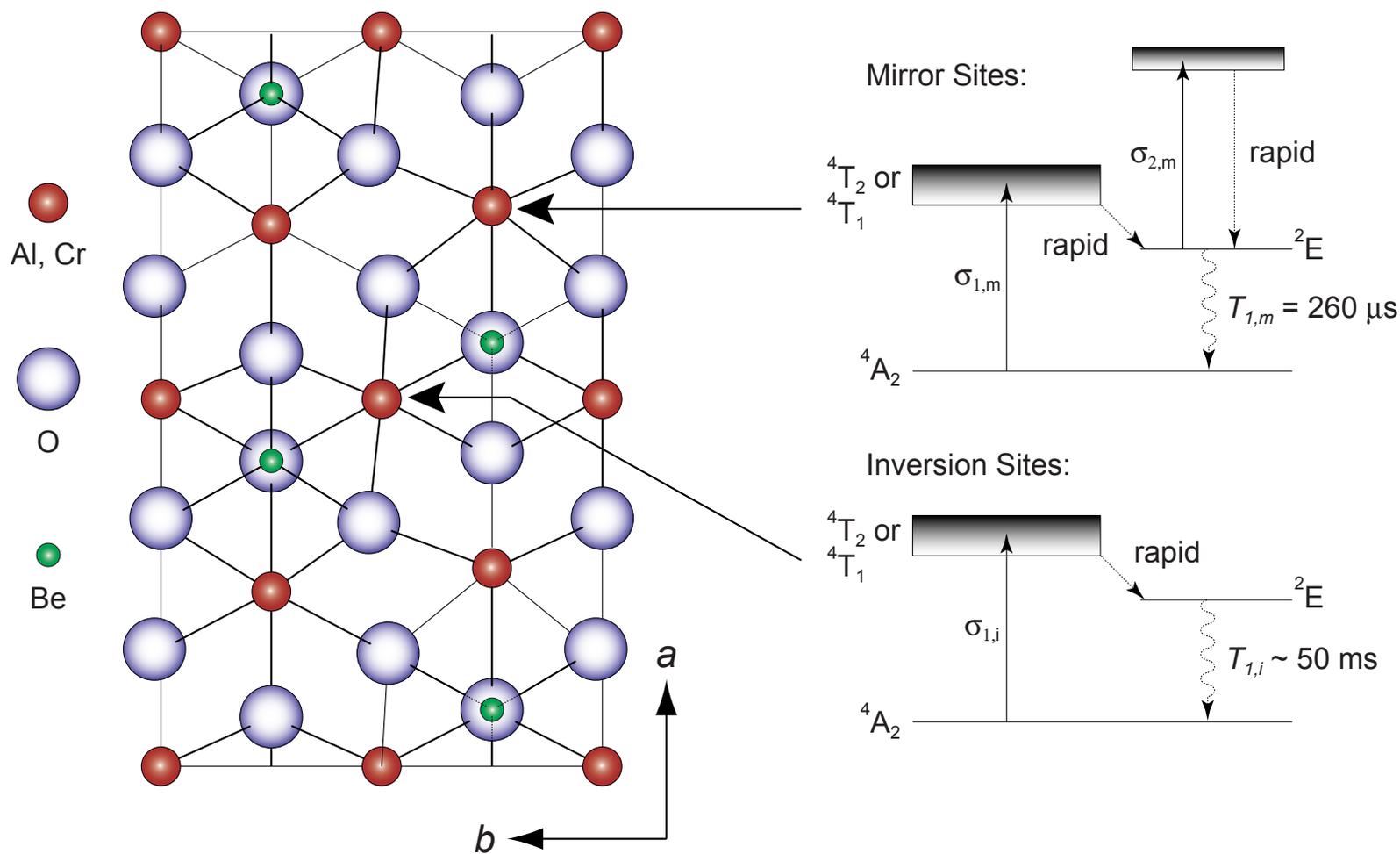
**Laser need not be frequency stabilized**

**Works with single beam (self-delayed)**

**Delay can be controlled through input intensity**

# Alexandrite Displays both Saturable and Reverse-Saturable Absorption

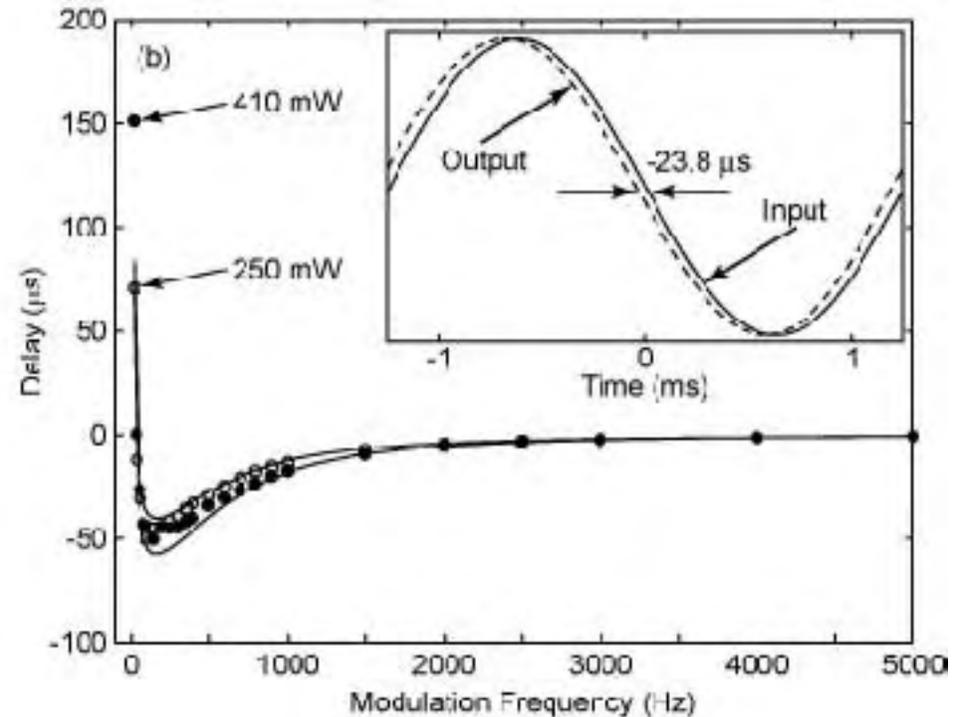
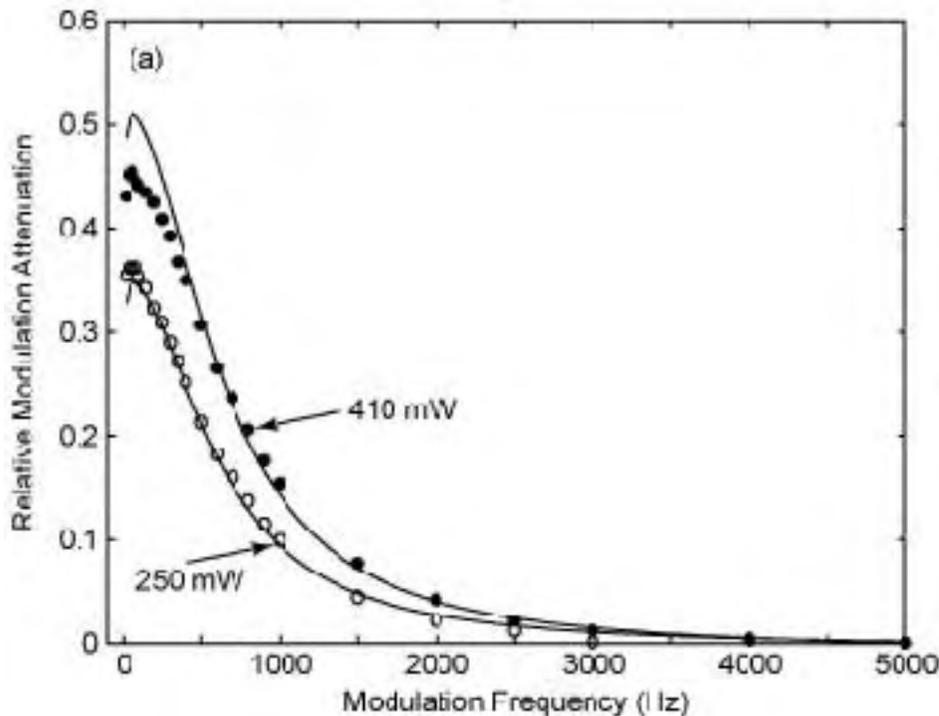
- Both slow and fast propagation observed in alexandrite



# Inverse-Saturable Absorption Produces Superluminal Propagation in Alexandrite

At 476 nm, alexandrite is an inverse saturable absorber

Negative time delay of 50  $\mu\text{s}$  corresponds to a velocity of -800 m/s



M. Bigelow, N. Lepeshkin, and RWB, Science, 2003

# Numerical Modeling of Pulse Propagation through Slow and Fast-Light Media

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Numerically integrate the reduced wave equation

$$\frac{\partial A}{\partial z} - \frac{1}{v_g} \frac{\partial A}{\partial t} = 0$$

and plot  $A(z,t)$  versus distance  $z$ .

Assume an input pulse with a Gaussian temporal profile.

Study three cases:

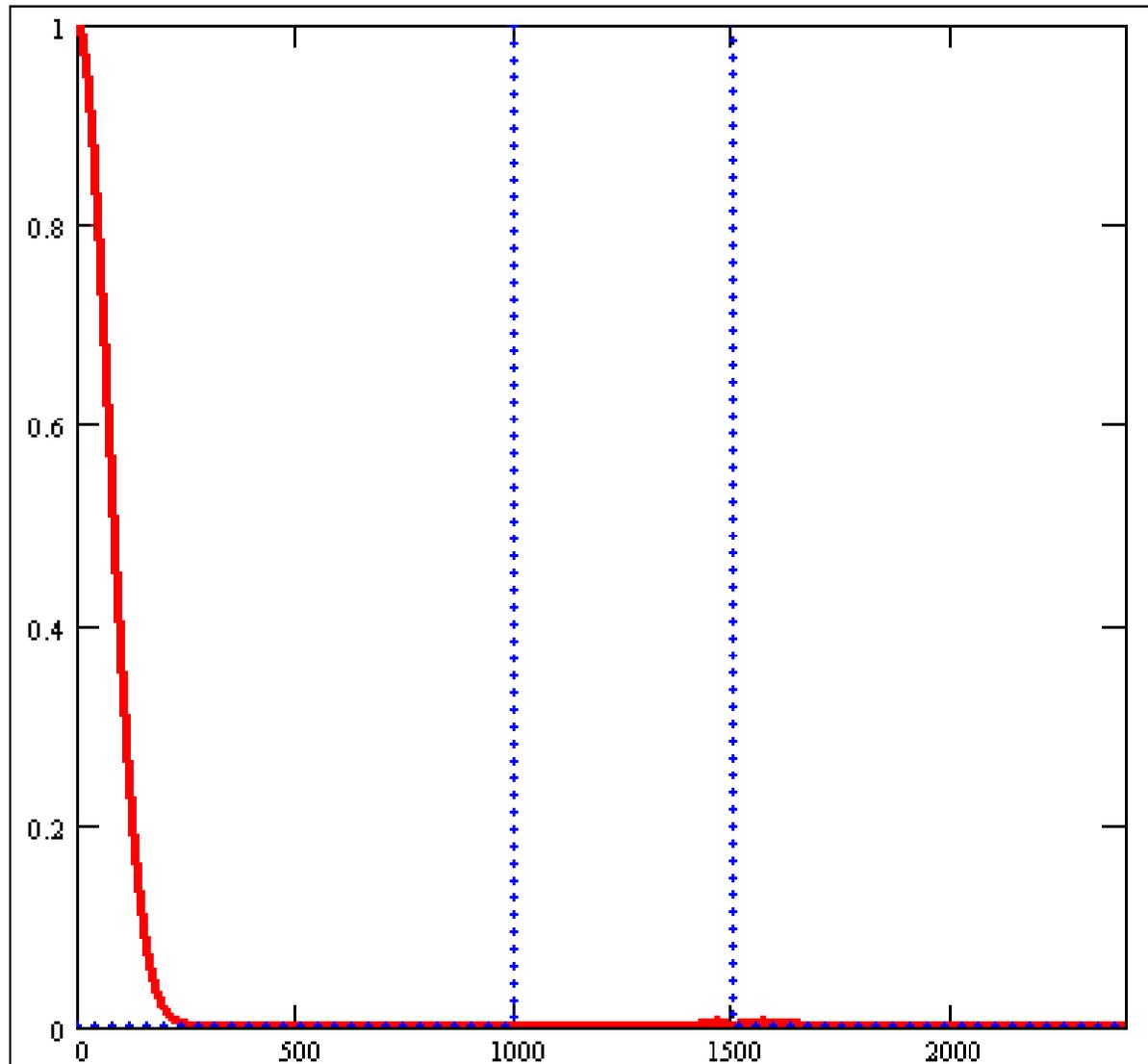
Slow light  $v_g = 0.5 c$

Fast light  $v_g = 5 c$  and  $v_g = -2 c$

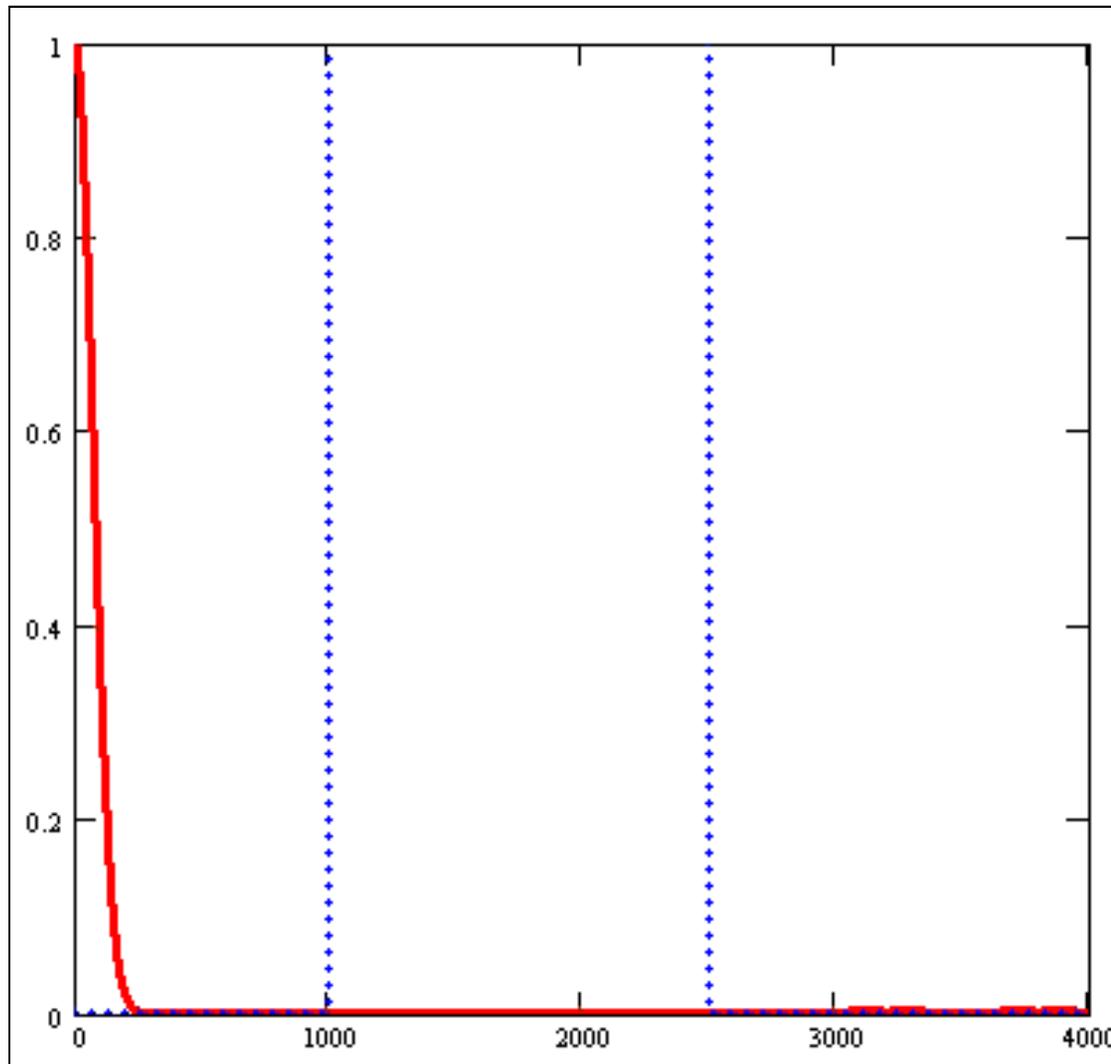
**CAUTION:** This is a very simplistic model. It ignores GVD and spectral reshaping.

See also Dogariu et al. Opt. Express 8, 344 (2001) and Milonni (2005).

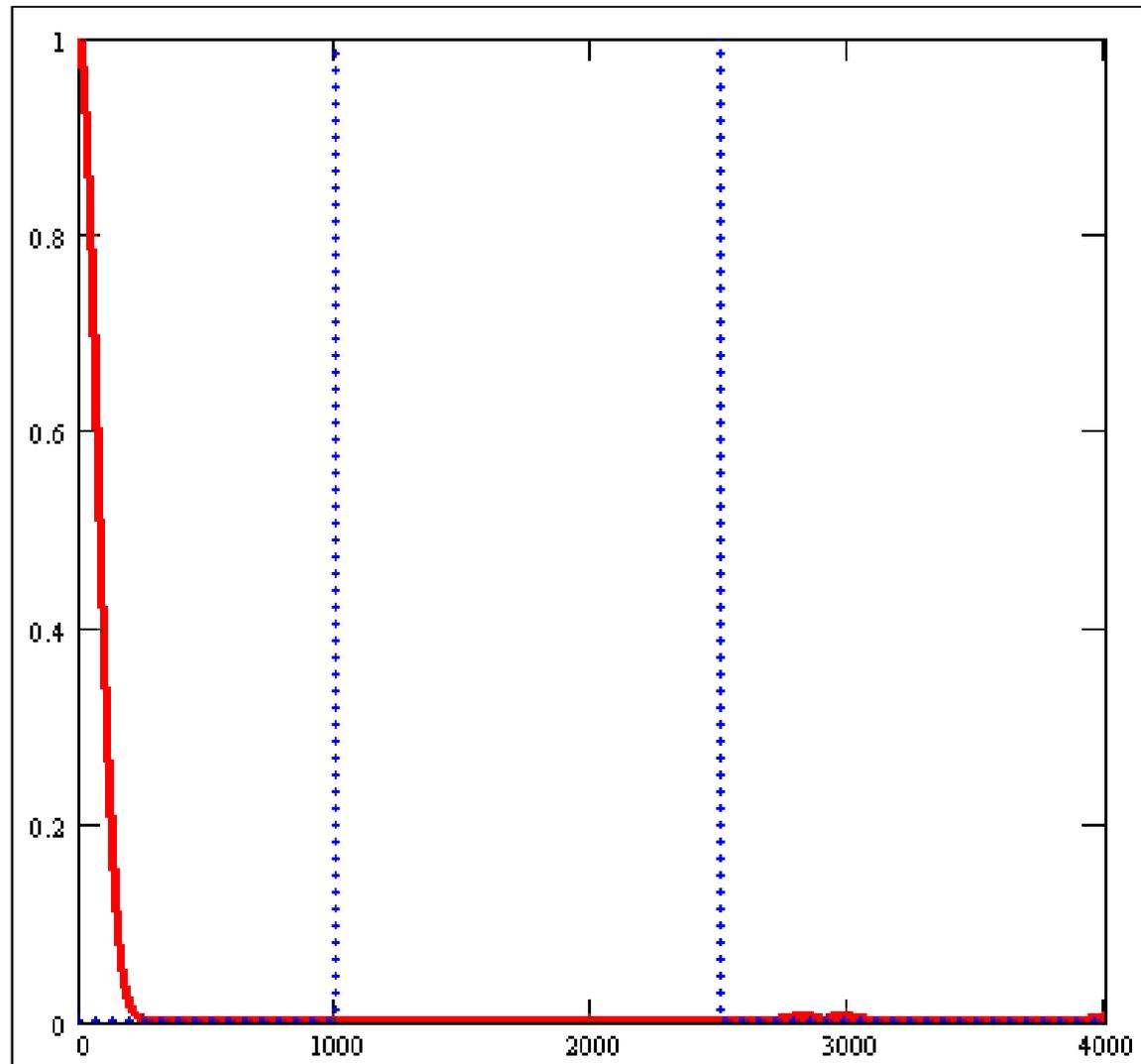
# Pulse Propagation through a Slow-Light Medium ( $n_g = 2$ , $v_g = 0.5 c$ )



# Pulse Propagation through a Fast-Light Medium ( $n_g = .2$ , $v_g = 5 c$ )

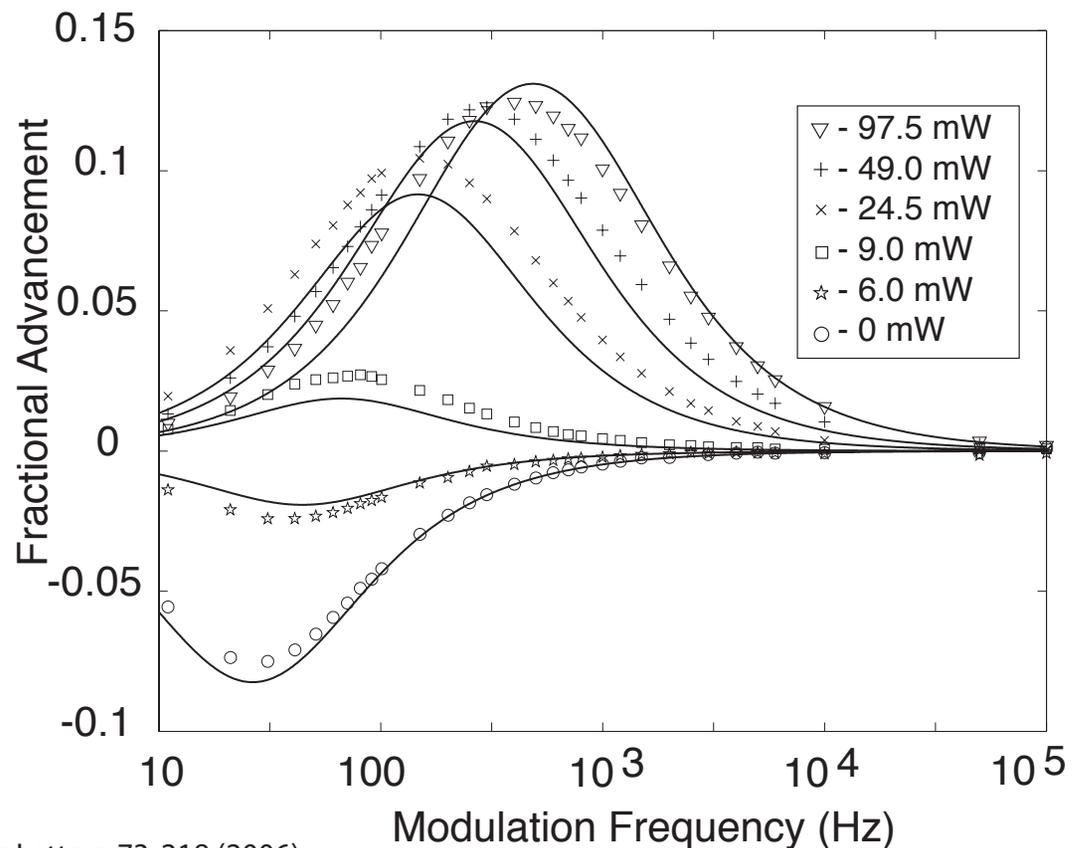
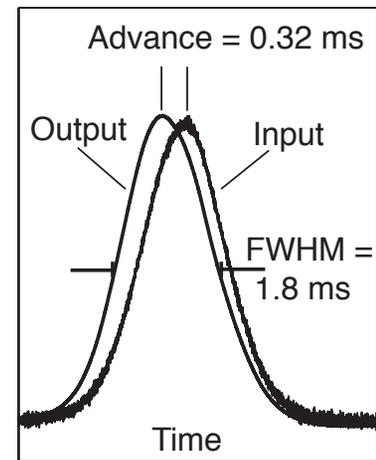
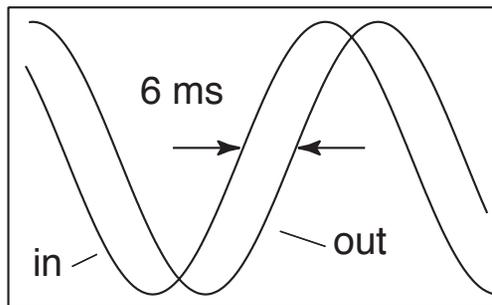
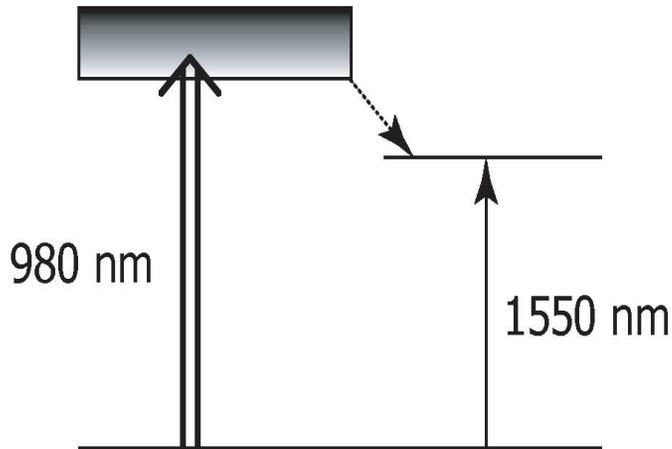


# Pulse Propagation through a Fast-Light Medium ( $n_g = -.5$ , $v_g = -2 c$ )

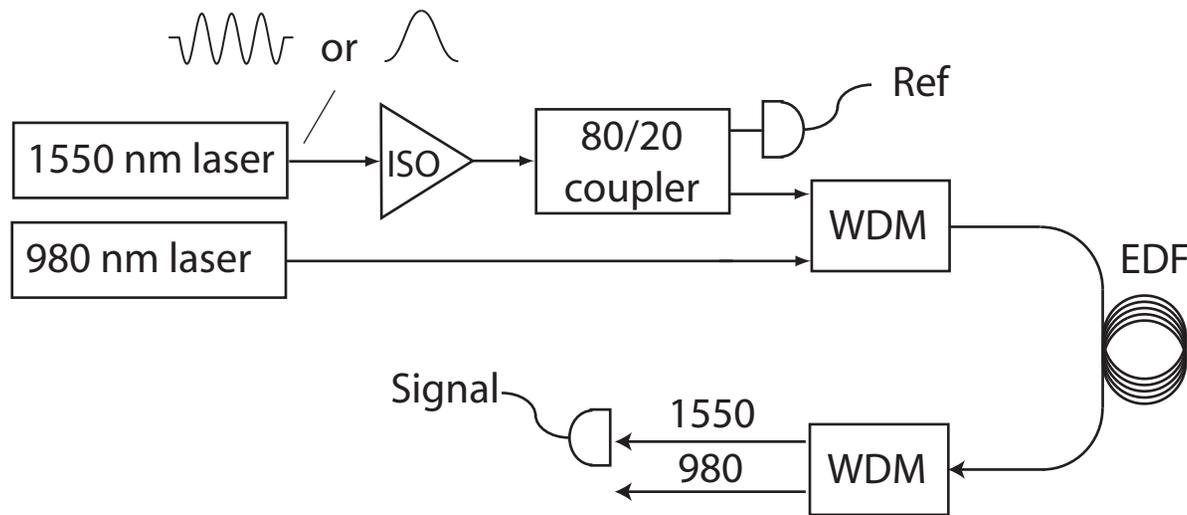


# Slow and Fast Light in an Erbium Doped Fiber Amplifier

- Fiber geometry allows long propagation length
- Saturable gain or loss possible depending on pump intensity



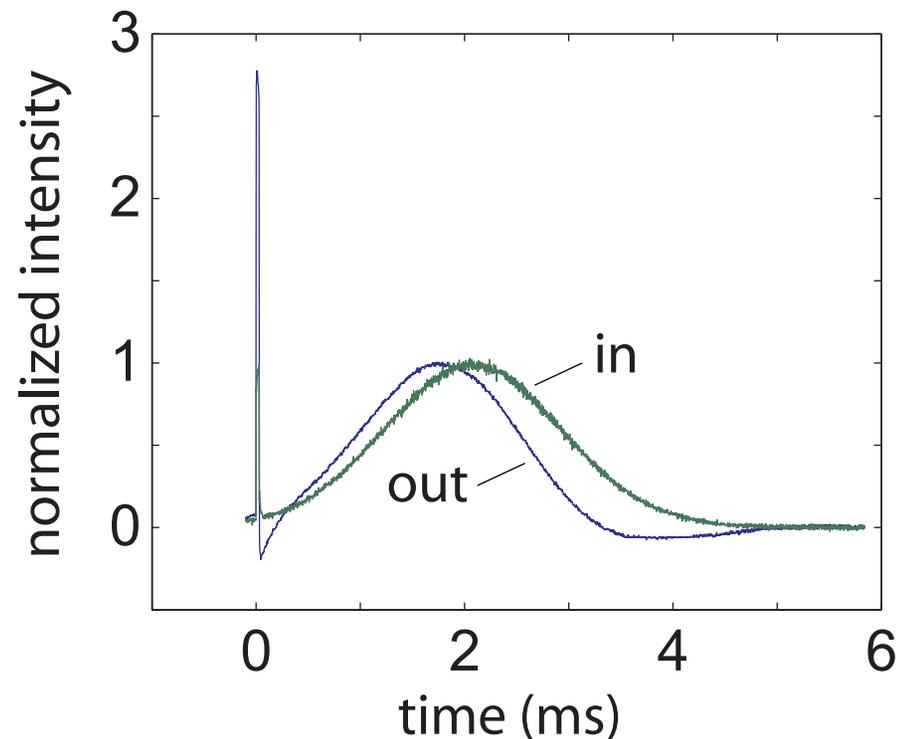
# Observation of Backward Pulse Propagation in an Erbium-Doped-Fiber Optical Amplifier



We time-resolve the propagation of the pulse as a function of position along the erbium-doped fiber.

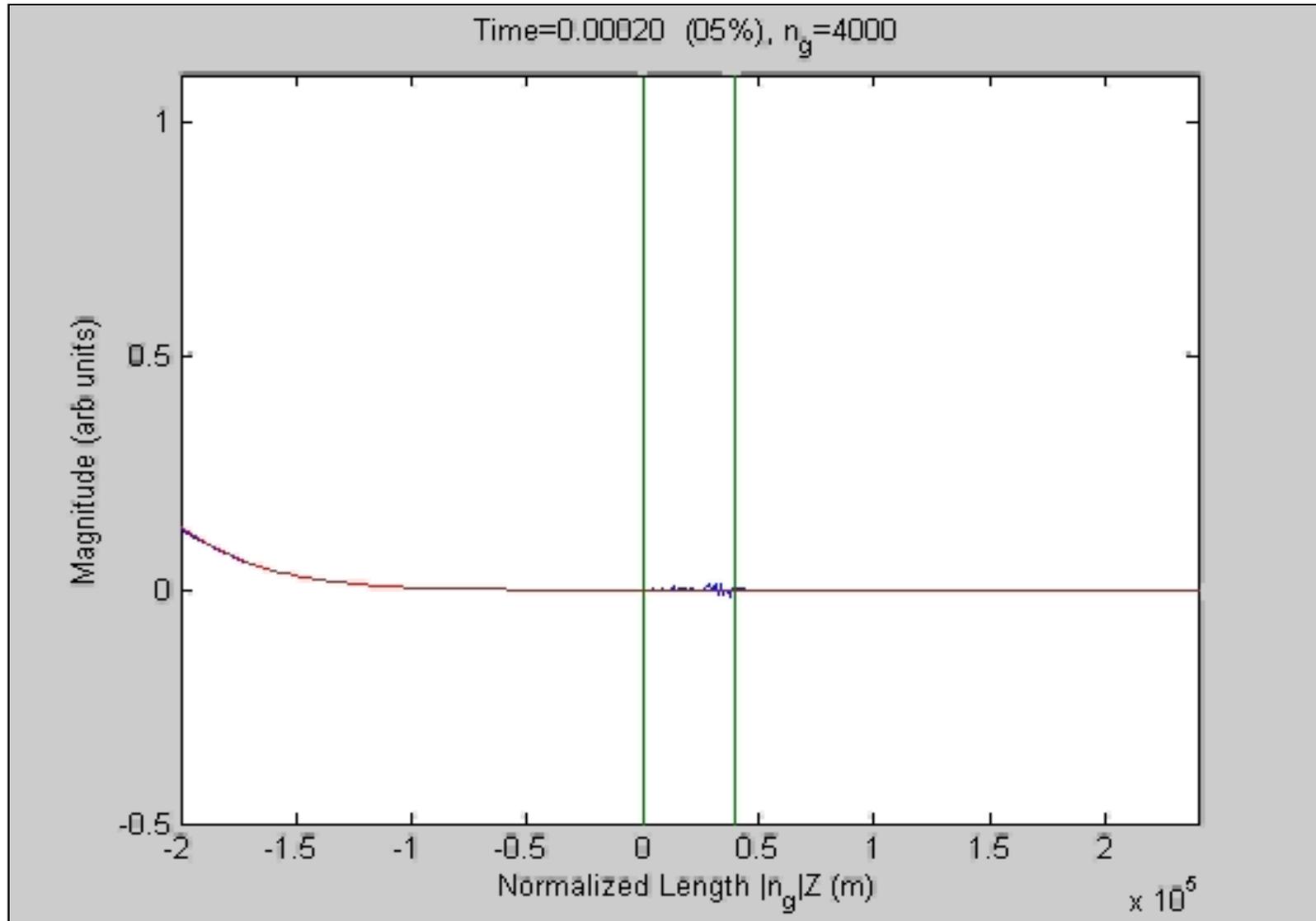
## Procedure

- cutback method
- couplers embedded in fiber



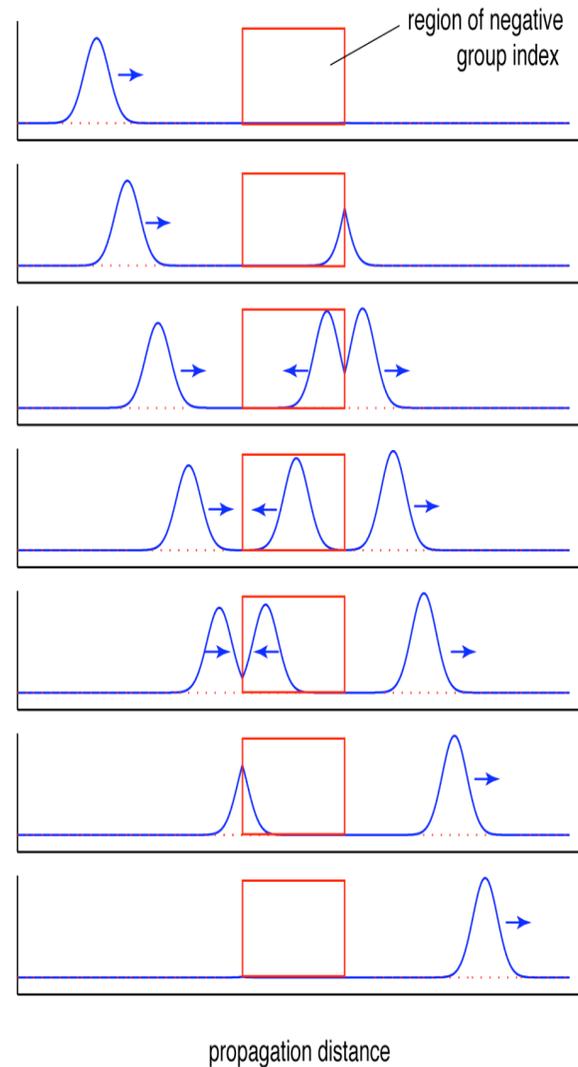
# Experimental Results: Backward Propagation in Erbium-Doped Fiber

Normalized: (Amplification removed numerically)

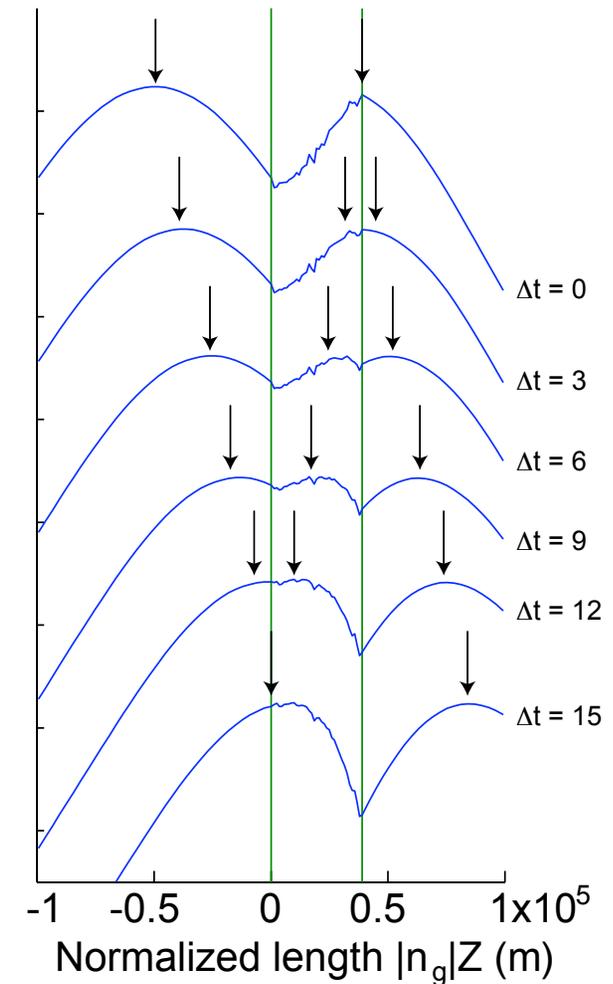


- A strongly counterintuitive phenomenon
- But entirely consistent with established physics
- G. M. Gehring, A. Schweinsberg, C. Barsi, N. Kostinski, and R. W. Boyd, *Science* 312, 985 2006.

## - conceptual prediction



## - laboratory results



# Observation of Backward Pulse Propagation in an Erbium-Doped-Fiber Optical Amplifier

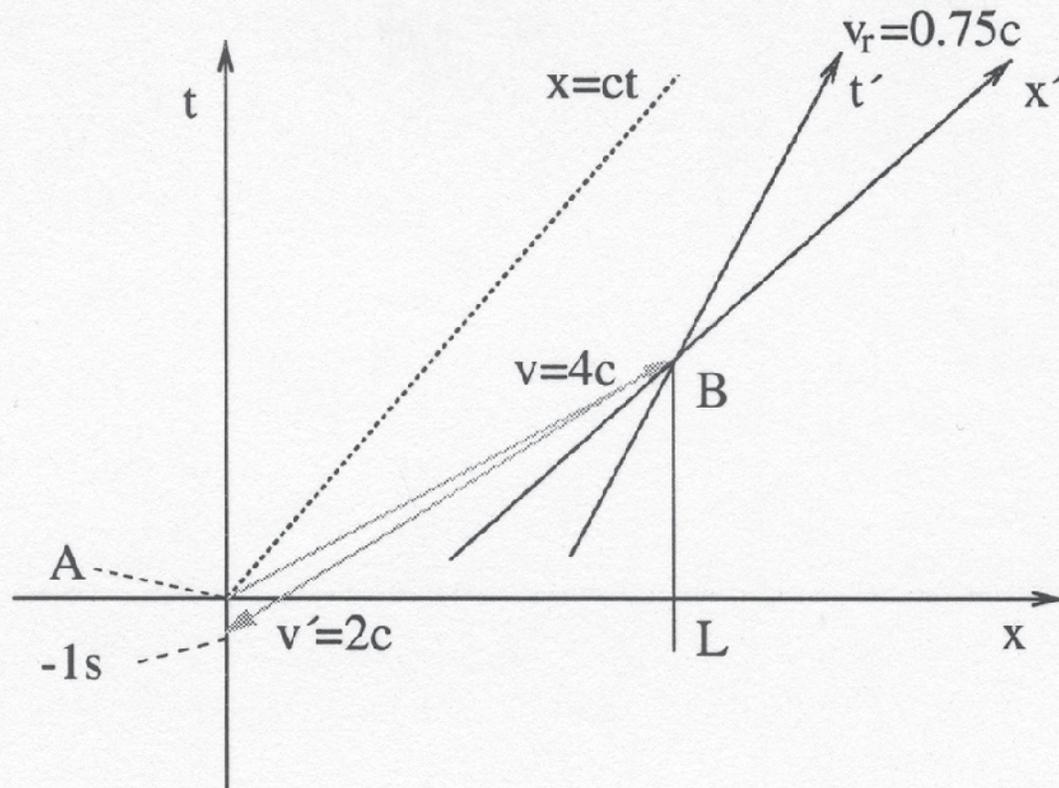
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Summary:

“Backwards” propagation is a realizable physical effect.

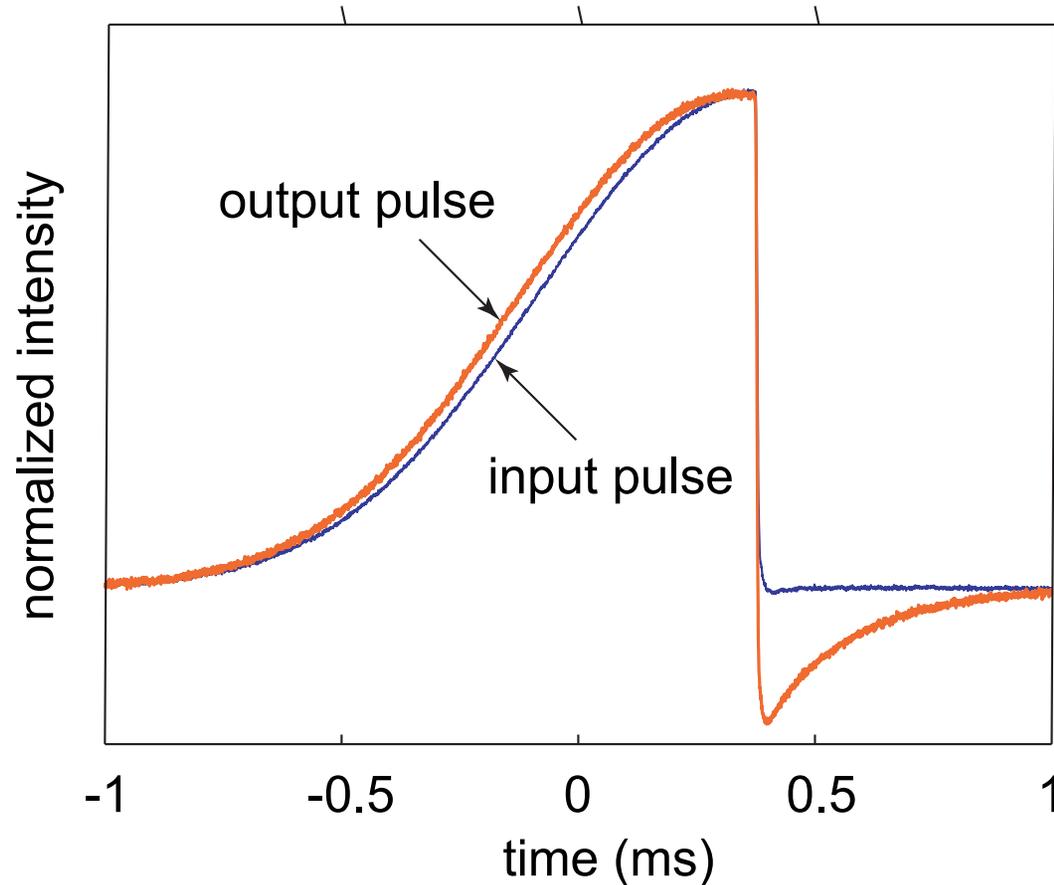
(Of course, many other workers have measured negative time delays. Our contribution was to measure the pulse evolution within the material medium.)

# Causality and Superluminal Signal Transmission



**Fig. 6** Coordinates of two inertial observers **A**  $(0,0)$  and **B** with  $O(x,t)$  and  $O'(x',t')$  moving with a relative velocity of  $0.75c$ . The distance  $L$  between **A** and **B** is  $2000000$  km. **A** makes use of a signal velocity  $v_s = 4c$  and **B** makes use of  $v'_s = 2c$ . The numbers in the example are chosen arbitrarily. The signal returns  $-1$  s in the past in **A**.

# Propagation of a Truncated Pulse through Alexandrite as a Fast-Light Medium



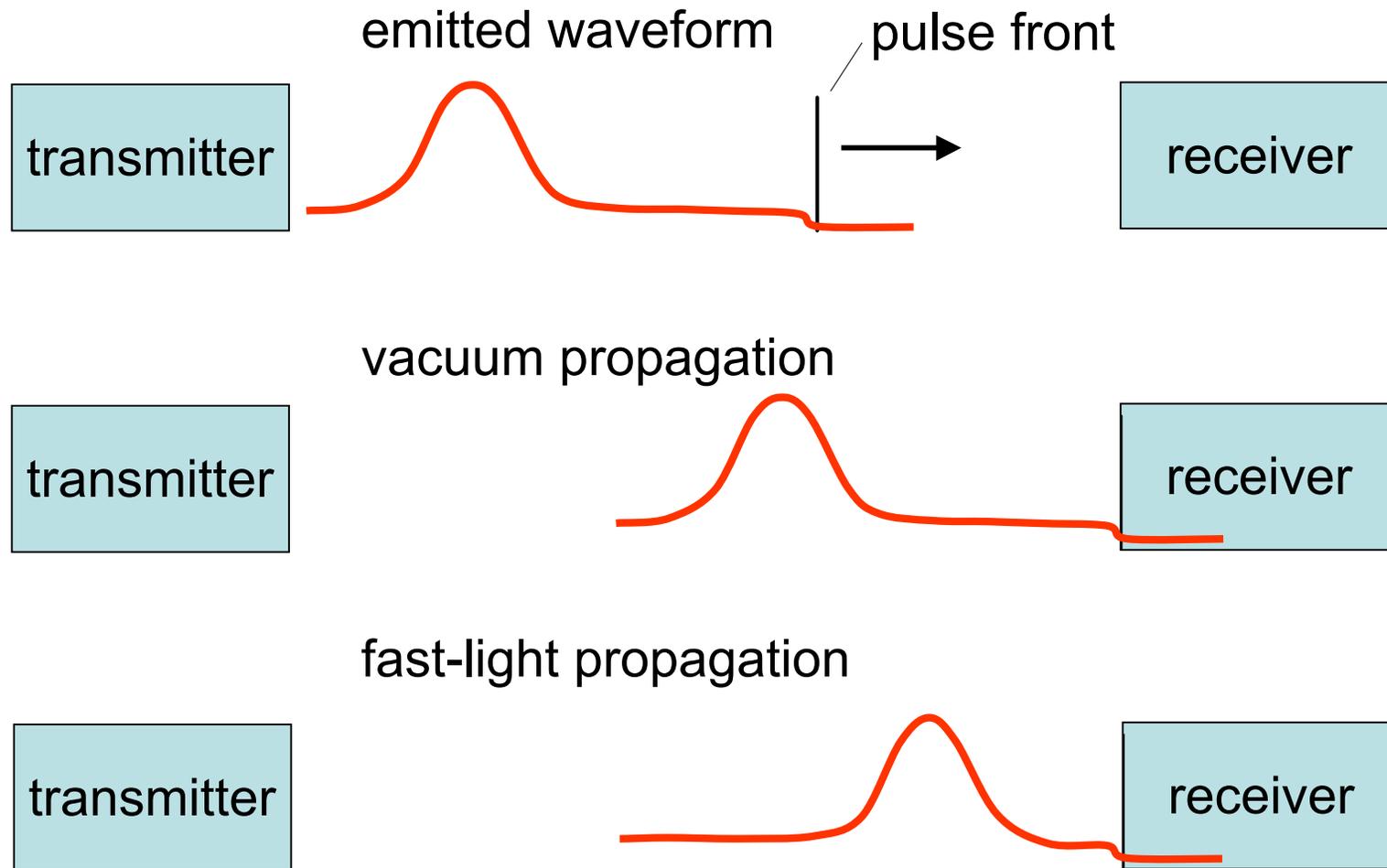
Smooth part of pulse propagates at group velocity  
Discontinuity propagates at phase velocity  
Information resides in points of discontinuity

Bigelow, Lepeshkin, Shin, and Boyd, *J. Phys: Condensed Matter*, 3117, 2006.

See also Stenner, Gauthier, and Neifeld, *Nature*, 425, 695, 2003.

# How to Reconcile Superluminality with Causality

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# Information Velocity – Tentative Conclusions

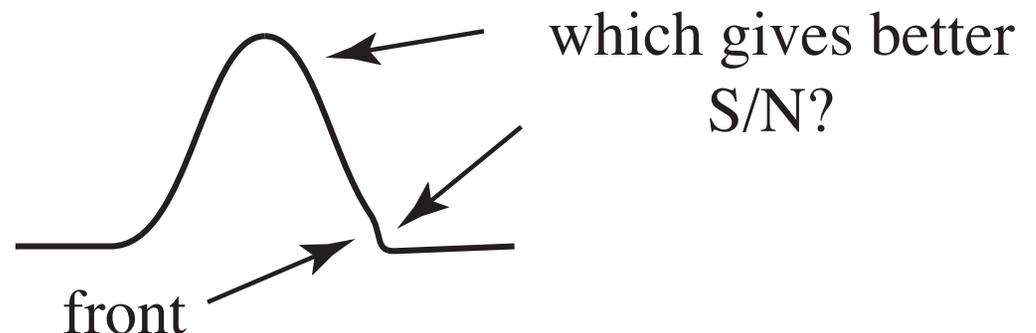
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In principle, the information velocity is equal to  $c$  for both slow- and fast-light situations. **So why is slow and fast light even useful?**

Because in many practical situations, we can perform reliable measurements of the information content only near the peak of the pulse.

In this sense, useful information often propagates at the group velocity.

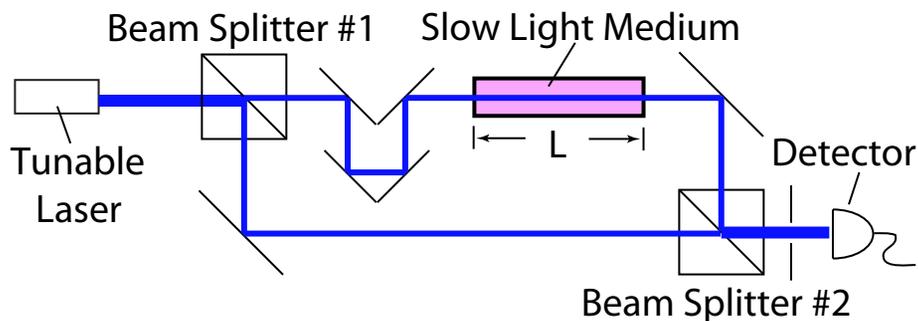
In a real communication system it would be really stupid to transmit pulses containing so much energy that one can reliably detect the very early leading edge of the pulse.



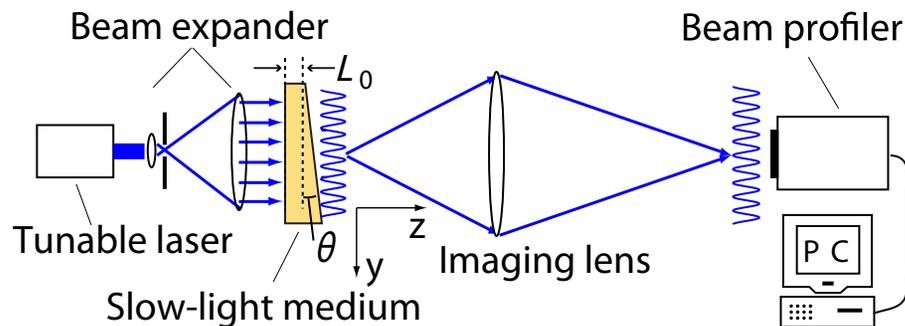
# Interferometry and Slow Light

- Under certain (but not all) circumstances, the sensitivity of an interferometer is increased by the group index of the material within the interferometer!
- Sensitivity of a spectroscopic interferometer is increased

Typical interferometer:



We use  $\text{CdS}_x\text{Se}_{1-x}$  as our slow-light medium

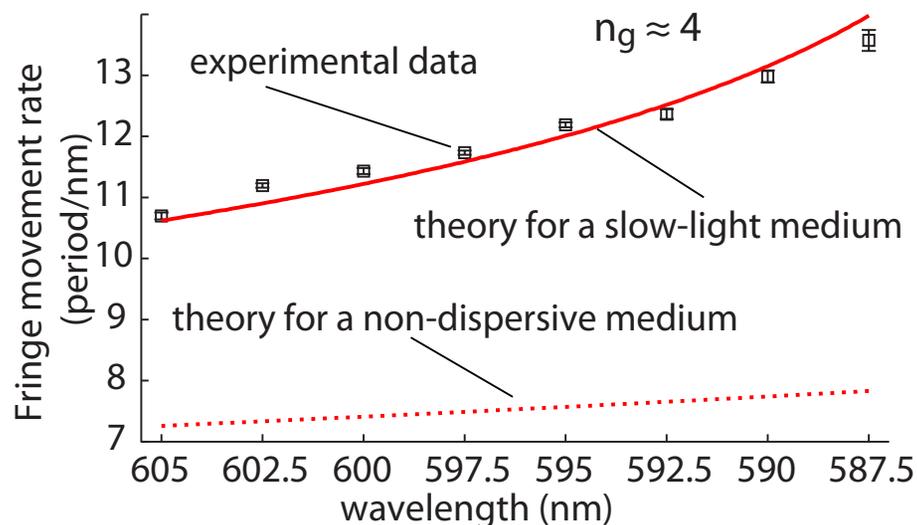


Here is why it works:

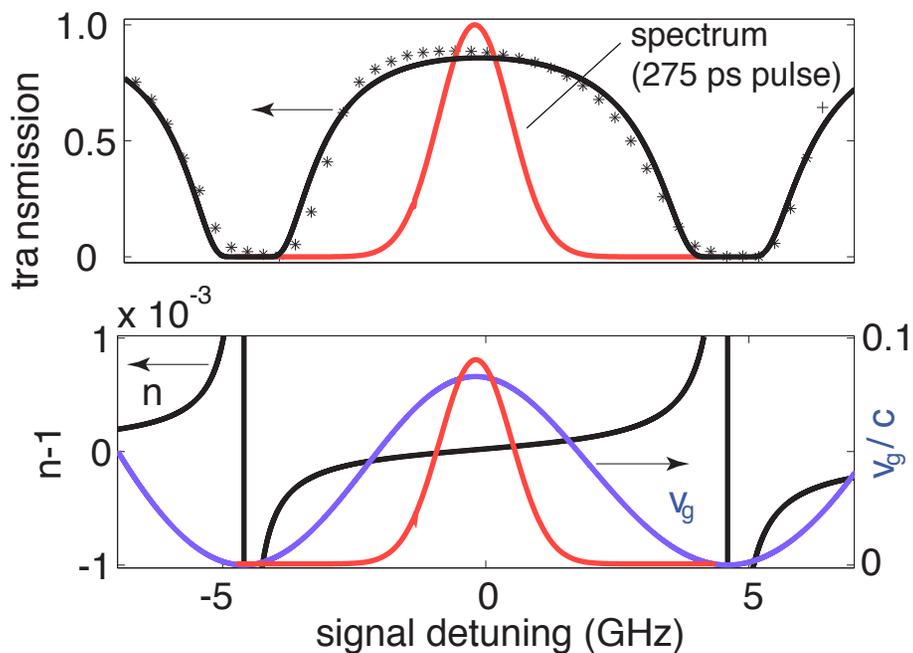
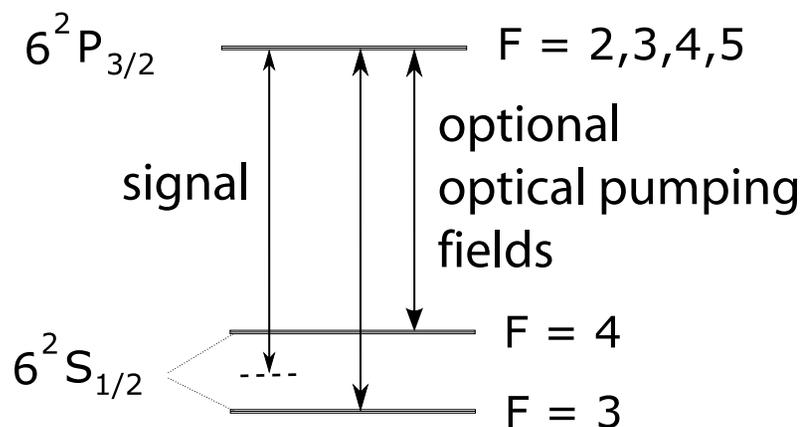
$$\frac{d\Delta\phi}{d\omega} = \frac{d}{d\omega} \left( \frac{\omega n L}{c} \right) = \frac{L}{c} \left( n + \omega \frac{dn}{d\omega} \right) = \frac{L n_g}{c}$$

Shi, Boyd, Gauthier, Dudley, Opt. Lett., 32, 915, 2007.

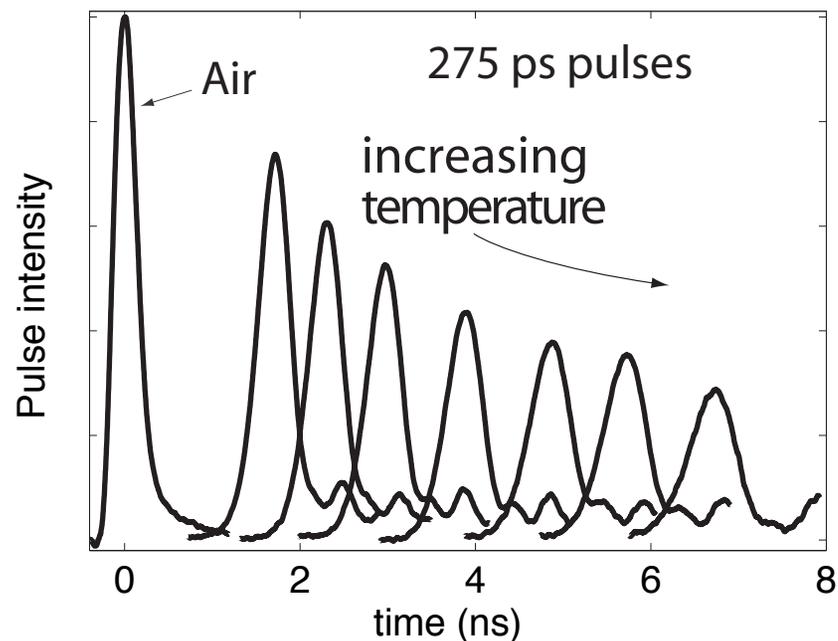
Our experimental results



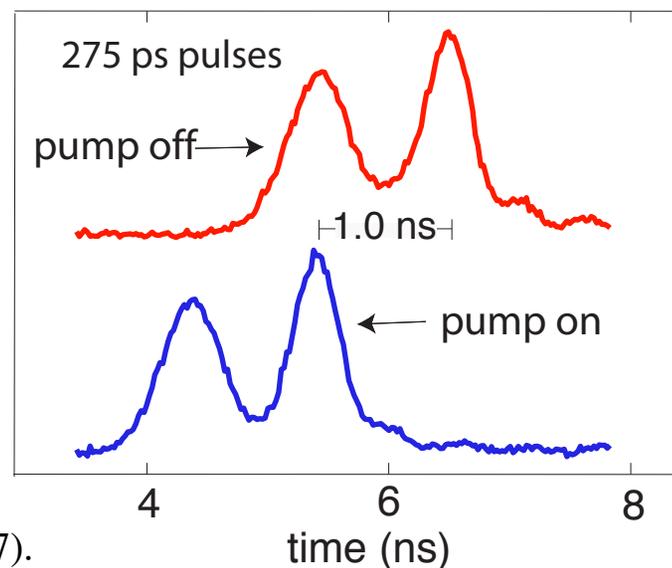
# Tunable Delays of up to 80 Pulse Widths in Atomic Cesium Vapor



- coarse tuning: temperature



- fine tuning: optical pumping



# How to Prevent Pulse Distortion (Which Can Limit Data Rates)

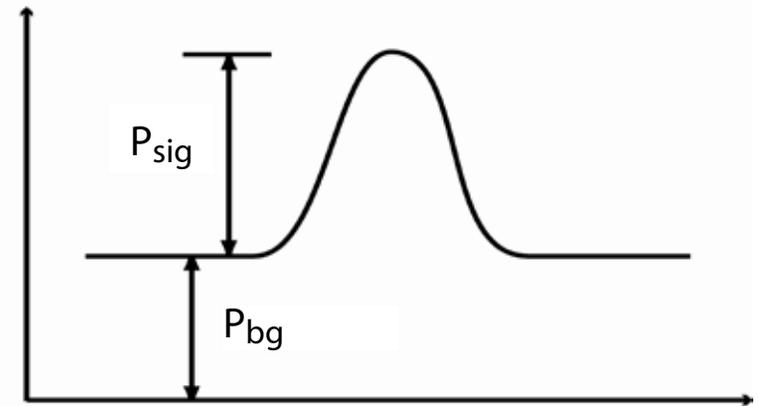
Two primary mechanisms for pulse distortion in EDFA

- Spectral broadening, leading to **temporal compression**  
CPO gain dip causes spectral components in the wings to be amplified more than central components

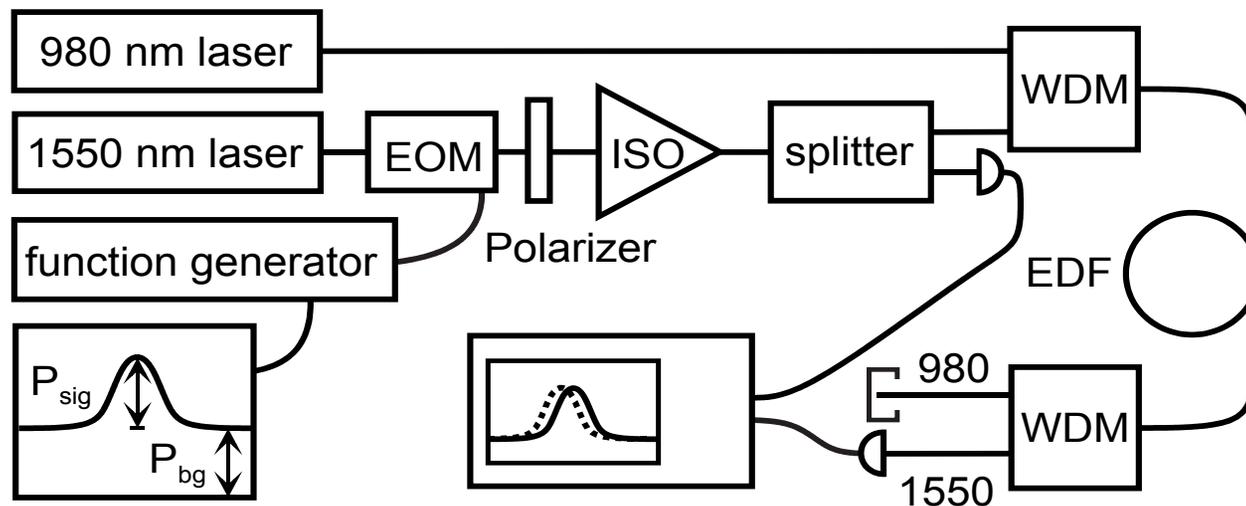
- Temporal gain recovery, leading to **temporal broadening**  
Leading edge of signal pulse saturates gain, but for long pulses, the trailing edge can experience recovered gain

To minimize second effect, add a cw background to reduce the influence of gain recovery

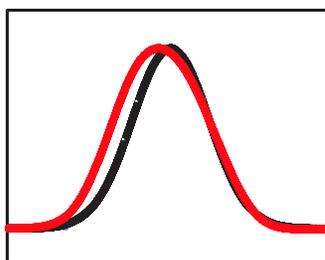
**For the proper choice of background power, the two effects exactly cancel!**



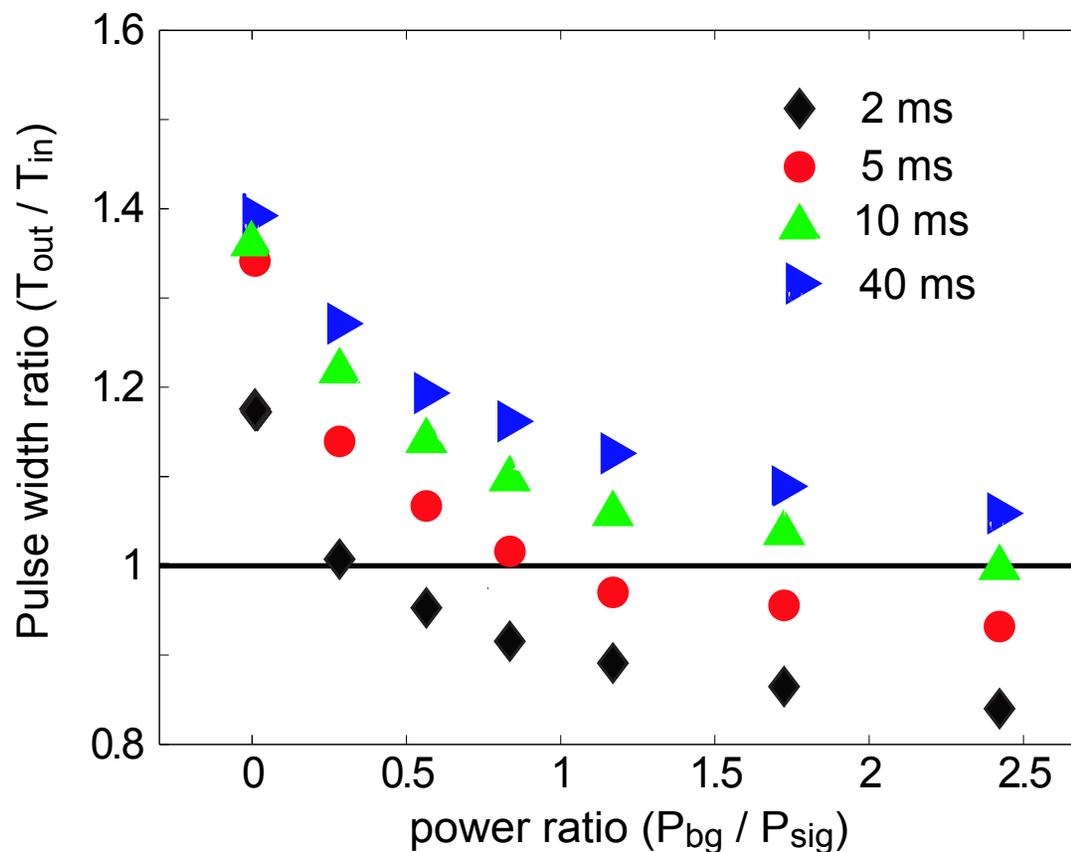
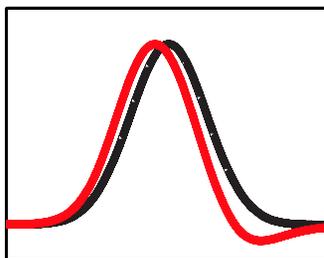
# Minimizing Pulse Distortion — Laboratory Results



broadening



compression



# Summary

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Slow-light techniques hold great promise for applications in telecommunications

Good progress being made in developing new slow-light techniques and applications

Backwards and superluminal propagation are strongly counterintuitive, but are fully explained by standard physics.

# Research in Quantum Imaging

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Can images be formed with higher resolution or better sensitivity through use of quantum states of light?

Can we "beat" the Rayleigh criterion?

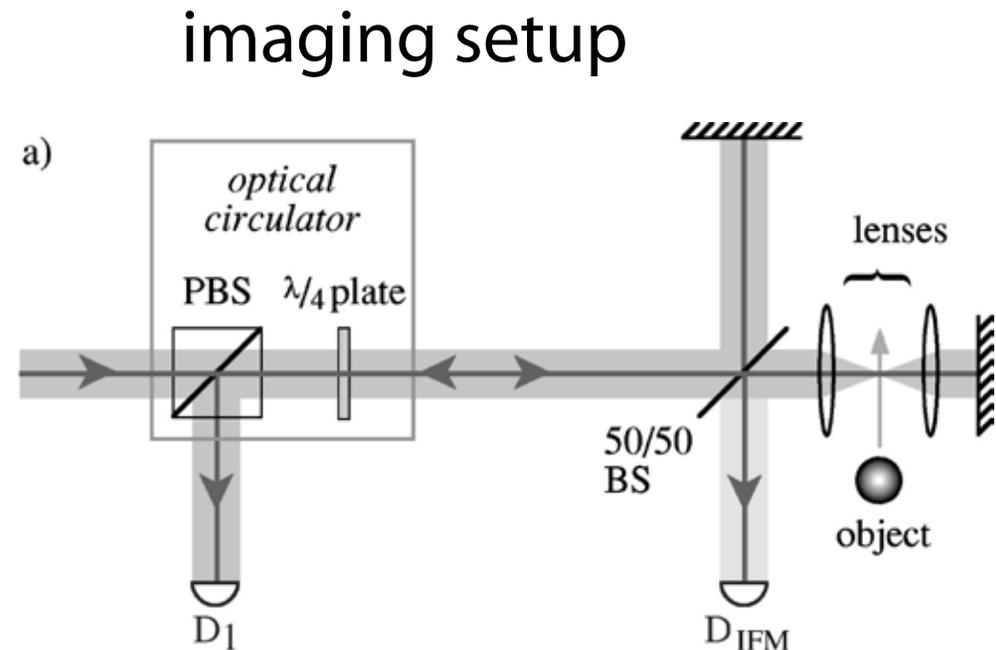
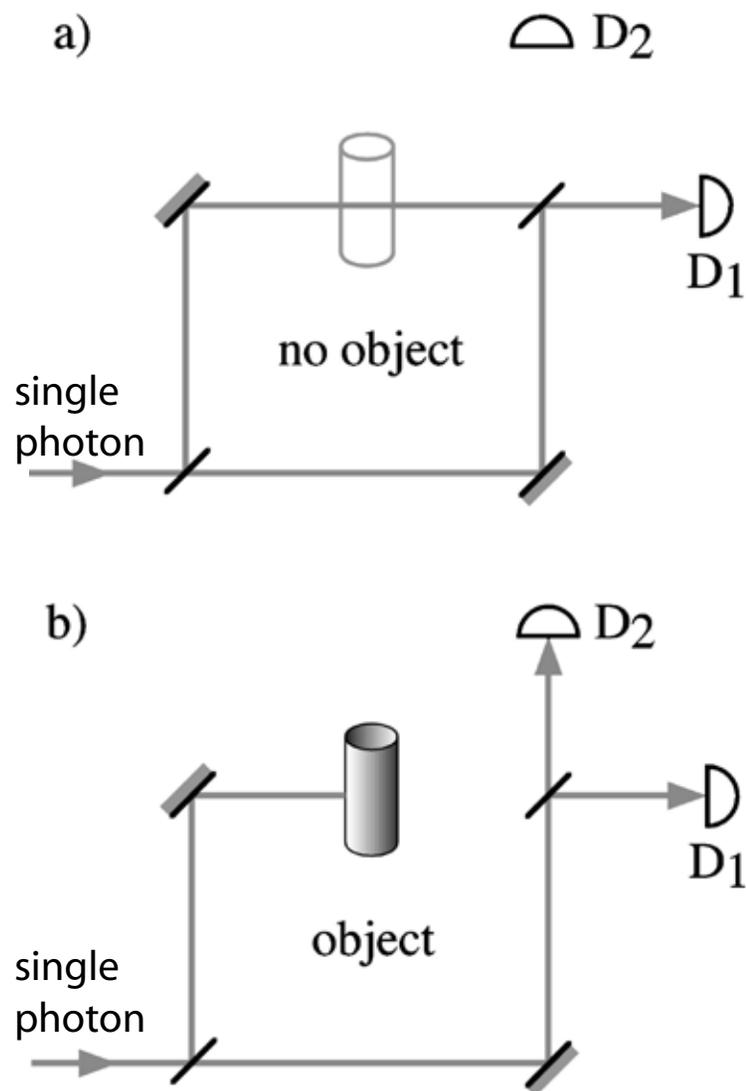
What are the implications of “interaction free” and “ghost” imaging

Quantum states of light: For instance, squeezed light or entangled beams of light.

Ghost and Interaction-Free Imaging

**Stealth Imaging**

# Quantum Imaging by Interaction-Free Measurement

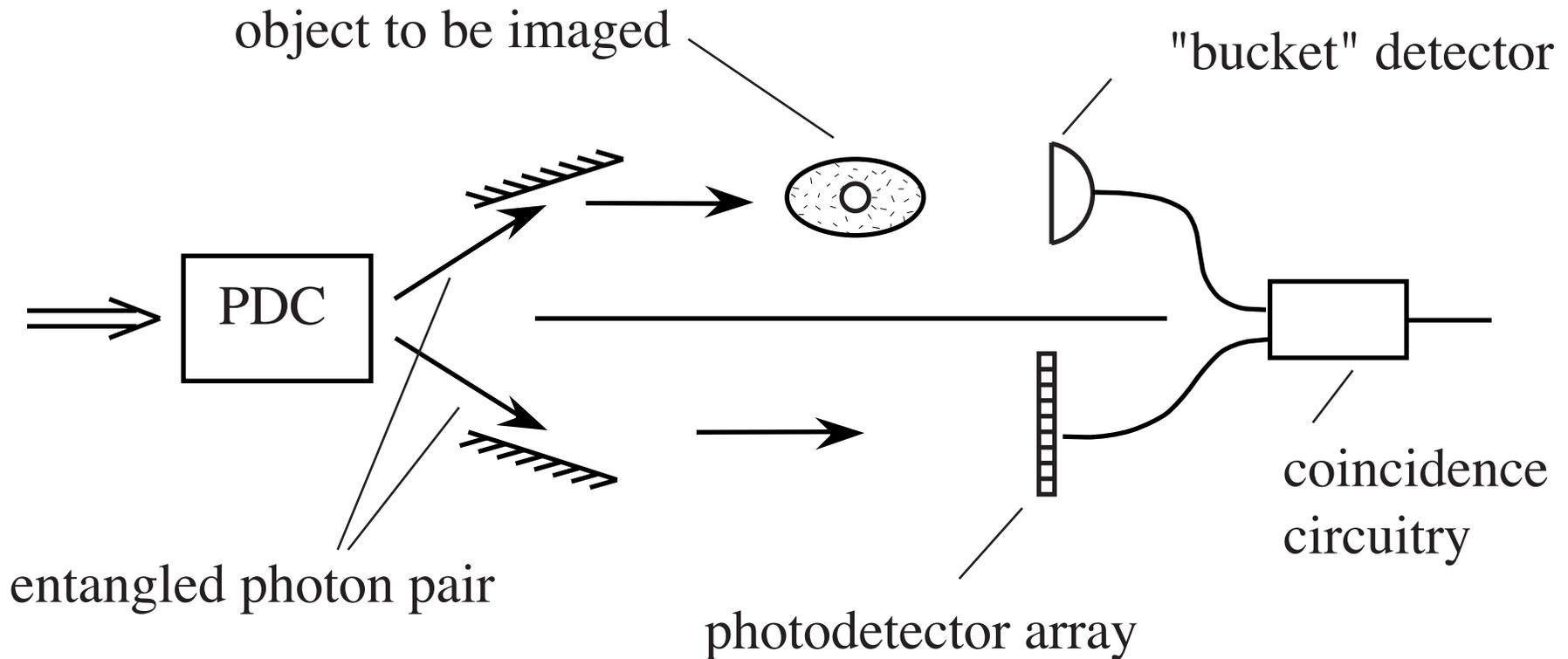


A. Elitzur and L. Vaidman, *Foundations of Physics*, 23 987 (1993).

Kwiat, Weinfurter, Herzog, Zeilinger, and Kasevich, *Phys. Rev. Lett.* 74 4763 1995

White, Mitchell, Nairz, and Kwiat, *Phys. Rev. A* 58, 605 (1998).

# Ghost (Coincidence) Imaging



- Obvious applicability to remote sensing!
- Is this a purely quantum mechanical process?

Strekalov et al., Phys. Rev. Lett. 74, 3600 (1995).

Pittman et al., Phys. Rev. A 52 R3429 (1995).

Abouraddy et al., Phys. Rev. Lett. 87, 123602 (2001).

Bennink, Bentley, and Boyd, Phys. Rev. Lett. 89 113601 (2002).

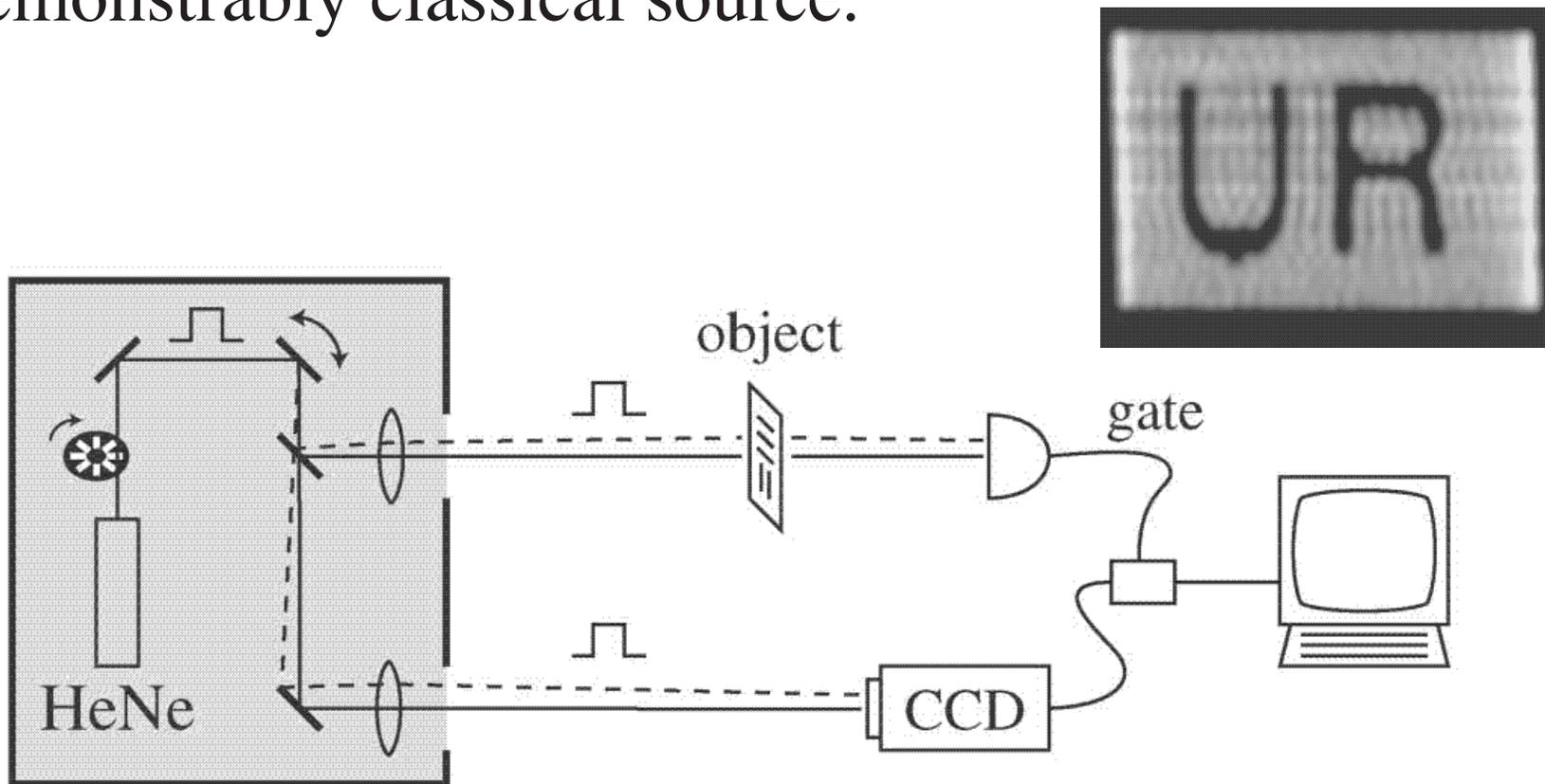
Bennink, Bentley, Boyd, and Howell, PRL 92 033601 (2004)

Gatti, Brambilla, and Lugiato, PRL 90 133603 (2003)

Gatti, Brambilla, Bache, and Lugiato, PRL 93 093602 (2003)

# Classical Coincidence Imaging

We have performed coincidence imaging with a demonstrably classical source.



Bennink, Bentley, and Boyd, Phys. Rev. Lett. **89** 113601 (2002).

# Further Development

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VOLUME 90, NUMBER 13

PHYSICAL REVIEW LETTERS

week ending  
4 APRIL 2003

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## Entangled Imaging and Wave-Particle Duality: From the Microscopic to the Macroscopic Realm

A. Gatti, E. Brambilla, and L. A. Lugiato

*INFN, Dipartimento di Scienze CC.FF.MM., Università dell'Insubria, Via Valleggio 11, 22100 Como, Italy*

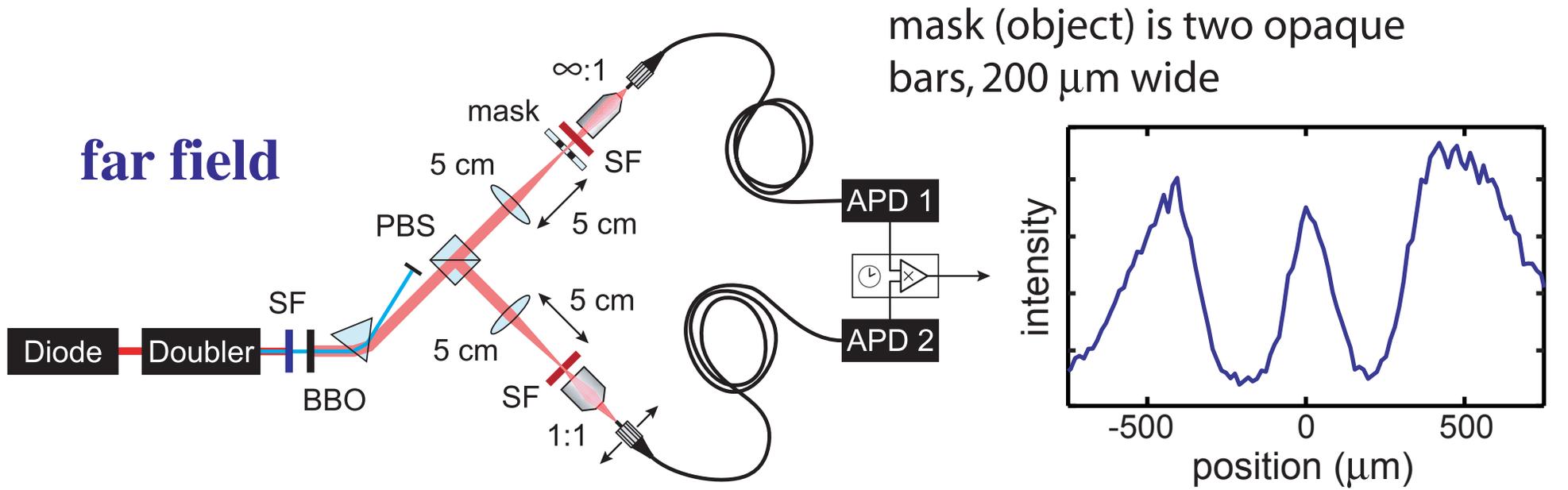
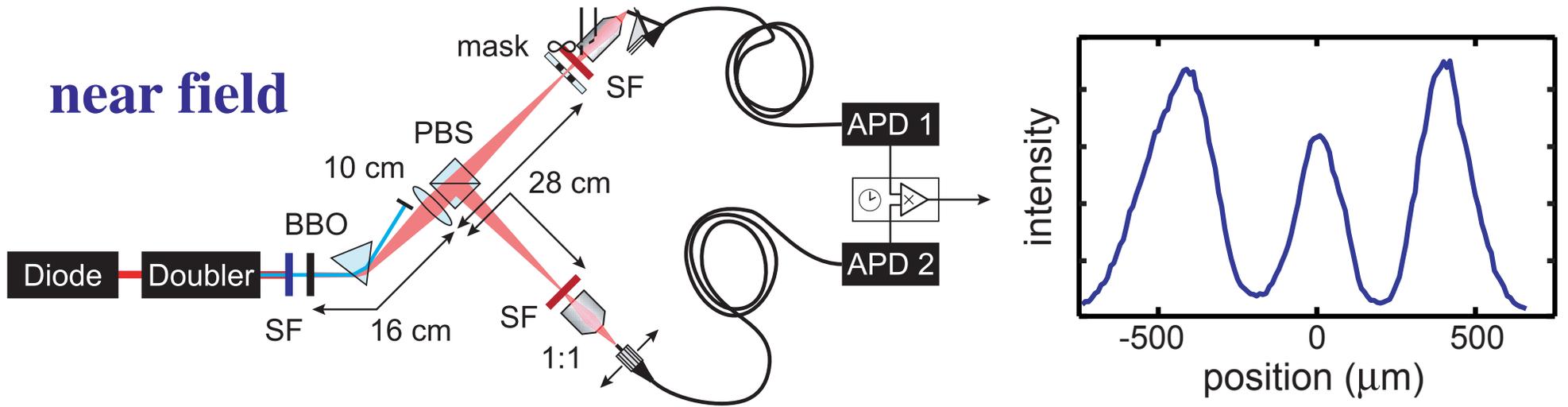
(Received 11 October 2002; published 3 April 2003)

We formulate a theory for ~~entangled~~ entangled imaging, which includes also the case of a large number of photons in the two entangled beams. We show that the results for imaging and for the wave-particle duality features, which have been demonstrated in the microscopic case, persist in the macroscopic domain. **We show that the quantum character of the imaging phenomena is guaranteed by the simultaneous spatial entanglement in the near and in the far field.**

DOI: 10.1103/PhysRevLett.90.133603

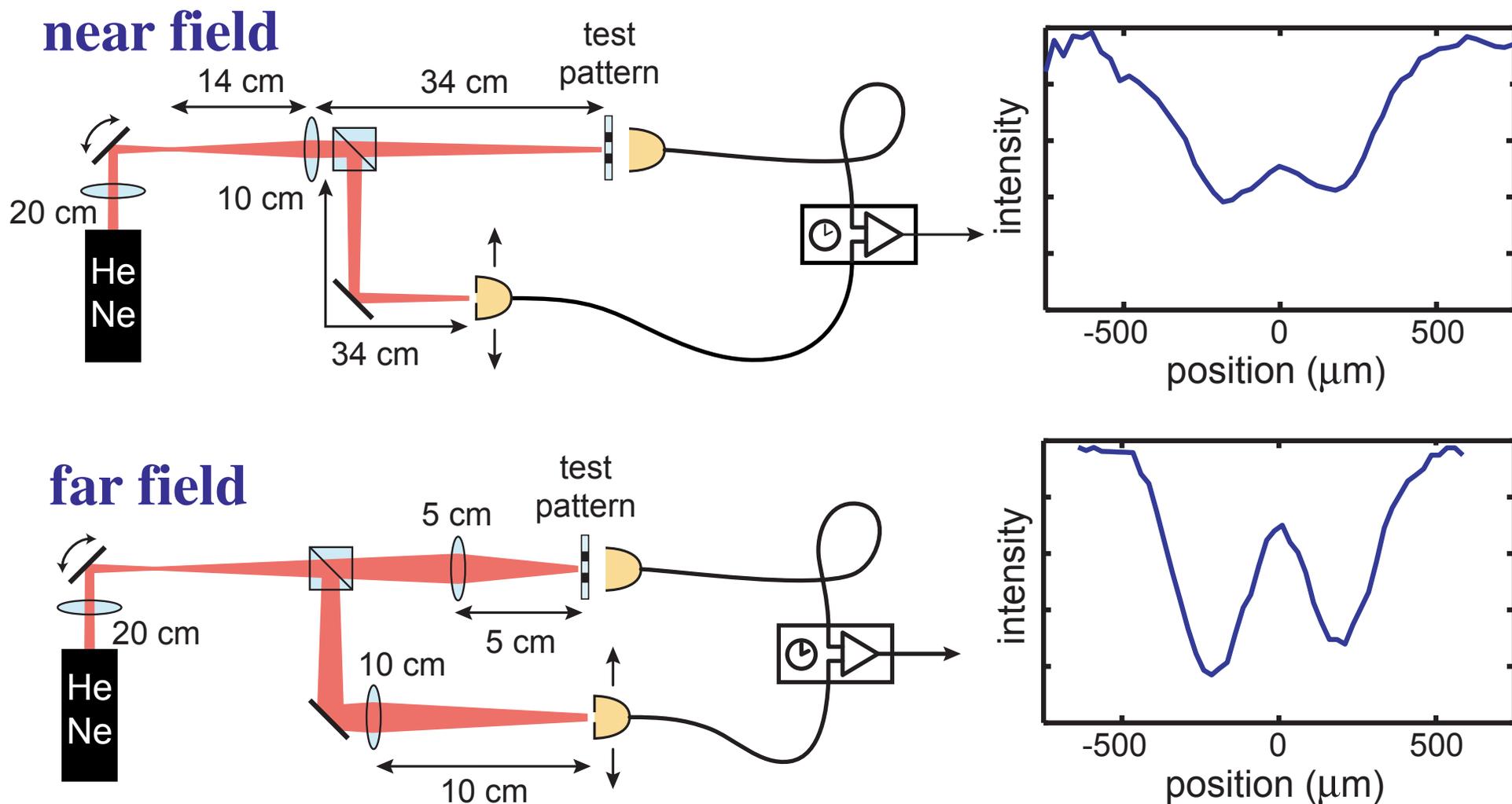
PACS numbers: 42.50.Dv, 03.65.Ud

# Near- and Far-Field Imaging Using Quantum Entanglement



Good imaging observed in both the near and far fields!

# Near- and Far-Field Imaging With a Classical Source



- Good imaging can be obtained only in near field **or** far field.
- Detailed analysis shows that in the quantum case the space-bandwidth exceeded the classical limit by a factor of ten.

# Is Entanglement Really Needed for Ghost Imaging with an Arbitrary Object Location?

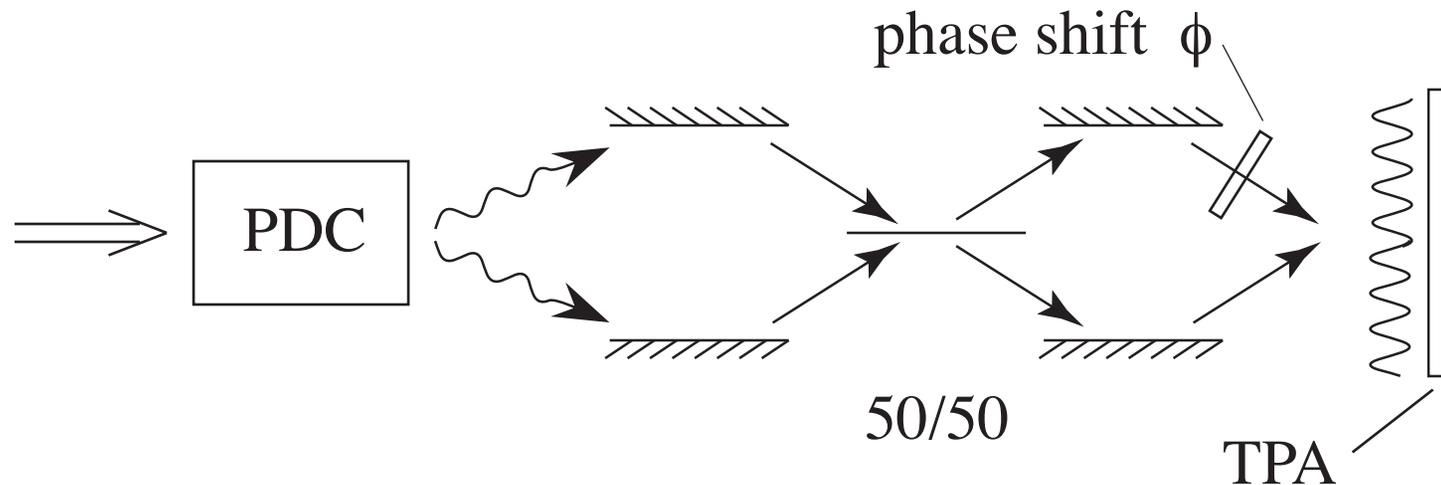
Gatti et al. (PRA and PRL, 2004) argue that thermal sources can mimic the quantum correlations produced by parametric down conversion. (Related to Brown-Twiss effect.)

Experimental confirmation of ghost imaging with thermal sources presented by Como and UMBC groups

But the contrast of the images formed in this manner is limited to  $1/2$  or  $1/N$  (depending on the circumstances) where  $N$  is the total number of pixels in the image.

# Quantum Lithography

- Entangled photons can be used to form an interference pattern with detail finer than the Rayleigh limit
- Process “in reverse” performs sub-Rayleigh microscopy, etc.
- Resolution  $\approx \lambda / 2N$ , where  $N$  = number of entangled photons



Boto et al., Phys. Rev. Lett. 85, 2733, 2000.

("al." includes Jon Dowling)

# Quantum Lithography: Easier Said Than Done

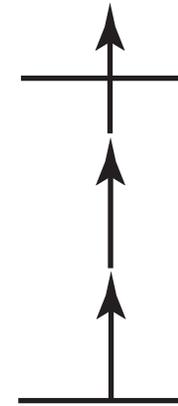
---

- Need an  $N$ -photon recording material

For proof-of-principle studies, can use  $N$ -th-harmonic generator, correlation circuitry,  $N$ -photon photodetector.

For actual implementation, use ????

Maybe best bet is UV lithographic material excited in the visible or a broad bandgap material such as PMMA excited by multiphoton absorption.



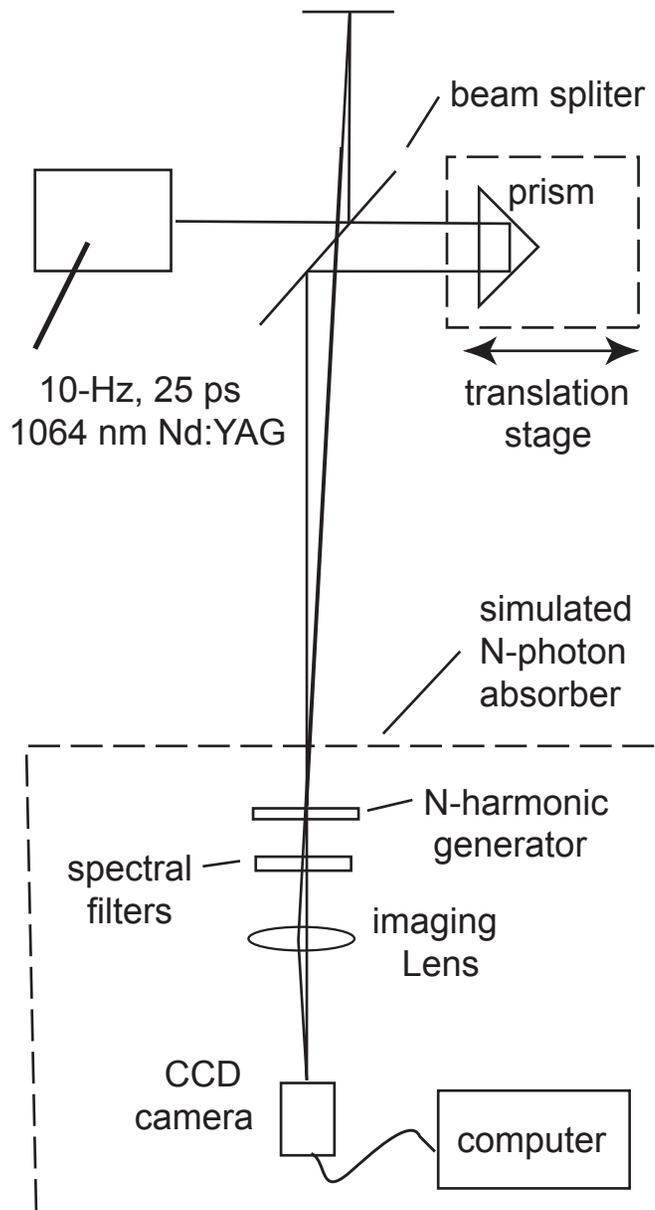
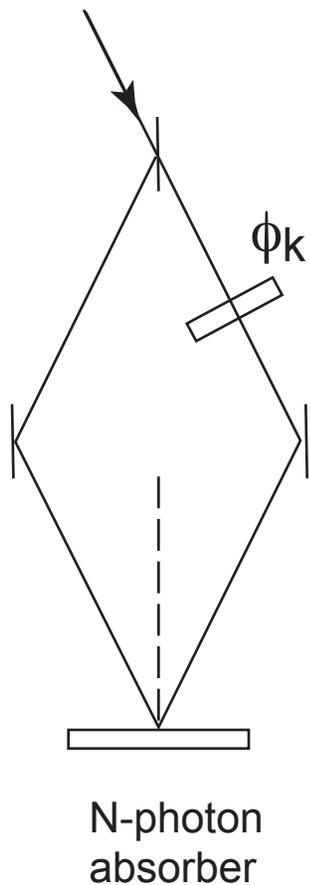
3PA in PMMA  
breaks chemical  
bond, modifying  
optical properties.  
Problem: self  
healing

- Need an intense source of individual biphotons (Inconsistency?)

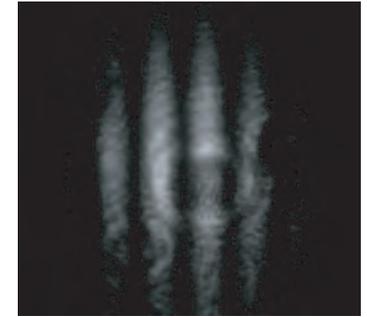
Maybe a high-gain OPA provides the best tradeoff between high intensity and required quantum statistics

# Classically Simulated (Non-Quantum) Quantum Lithography

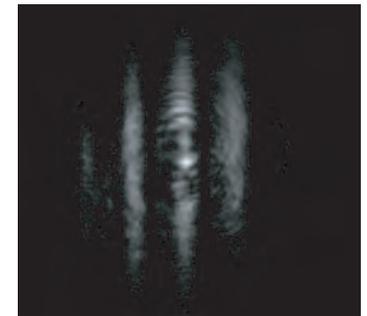
Concept: average  $M$  shots with the phase of shot  $k$  given by  $2\pi k/M$



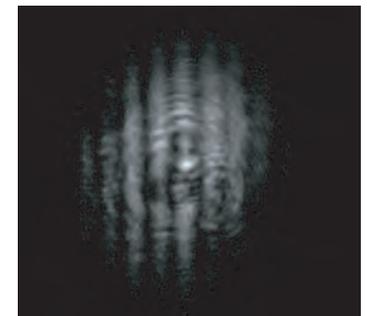
One-photon absorber  
( $N=1, M=1$ )



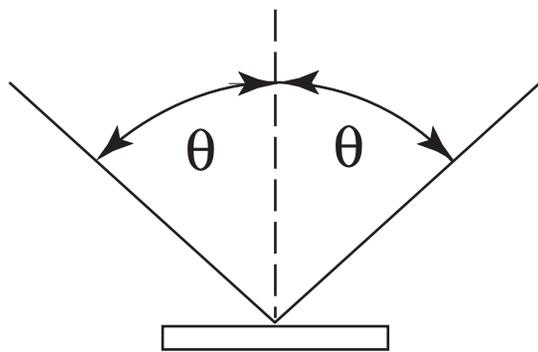
Two-photon absorber  
( $N=2, M=1$ )



Two-photon absorber  
two exposures  
( $N=2, M=2$ )



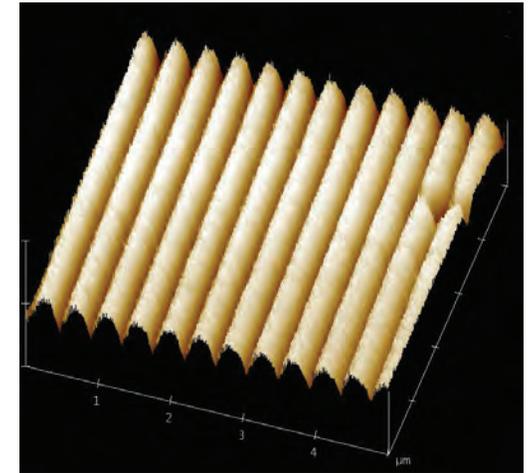
# Demonstration of Fringes Written into PMMA



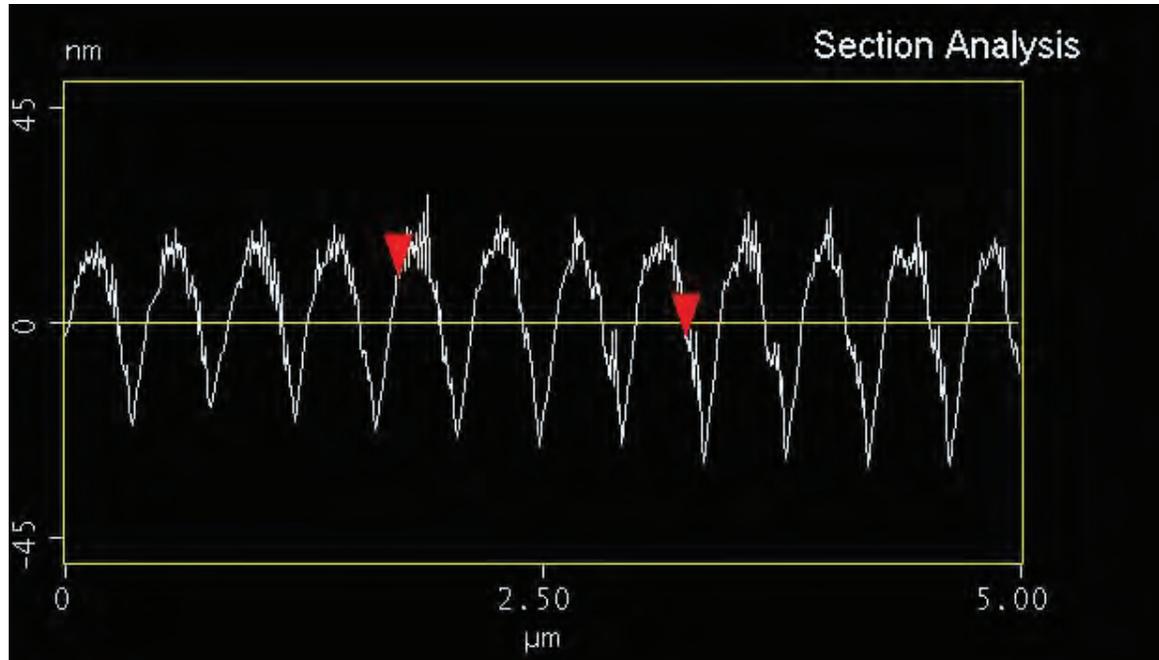
N-photon absorber  
(N = 3 ?)

$\theta = 70$  degrees  
write wavelength = 800 nm  
pulse energy = 130  $\mu$ J per beam  
pulse duration = 120 fs  
period =  $\lambda / (2 \sin \theta) = 425$  nm

PMMA on glass substrate  
develop for 10 sec in MBIK  
rinse 30 sec in deionized water



AFM



PMMA is a standard lithographic material

# Summary

Quantum lithography has a good chance of becoming a reality.

The quantum vs. classical nature of ghost imaging is more subtle than most of us had appreciated.

Many of our cherished “quantum effects” can be mimicked classically.

There is still work to be done in the context of quantum imaging to delineate the quantum/classical frontier.

# Special Thanks to My Students and Research Associates



**Thank you for your attention!**



# Physics is all about asking the right questions

Just ask

Evelyn **Hu**

**Watt** Webb (or James **Watt**)

Michael **Ware**

**Wen I** Wang

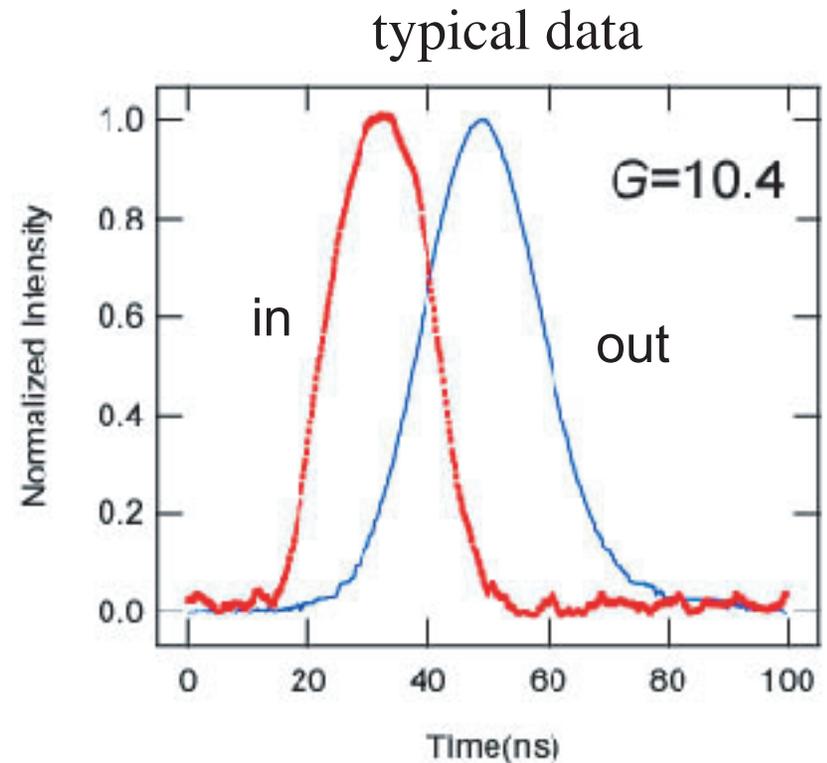
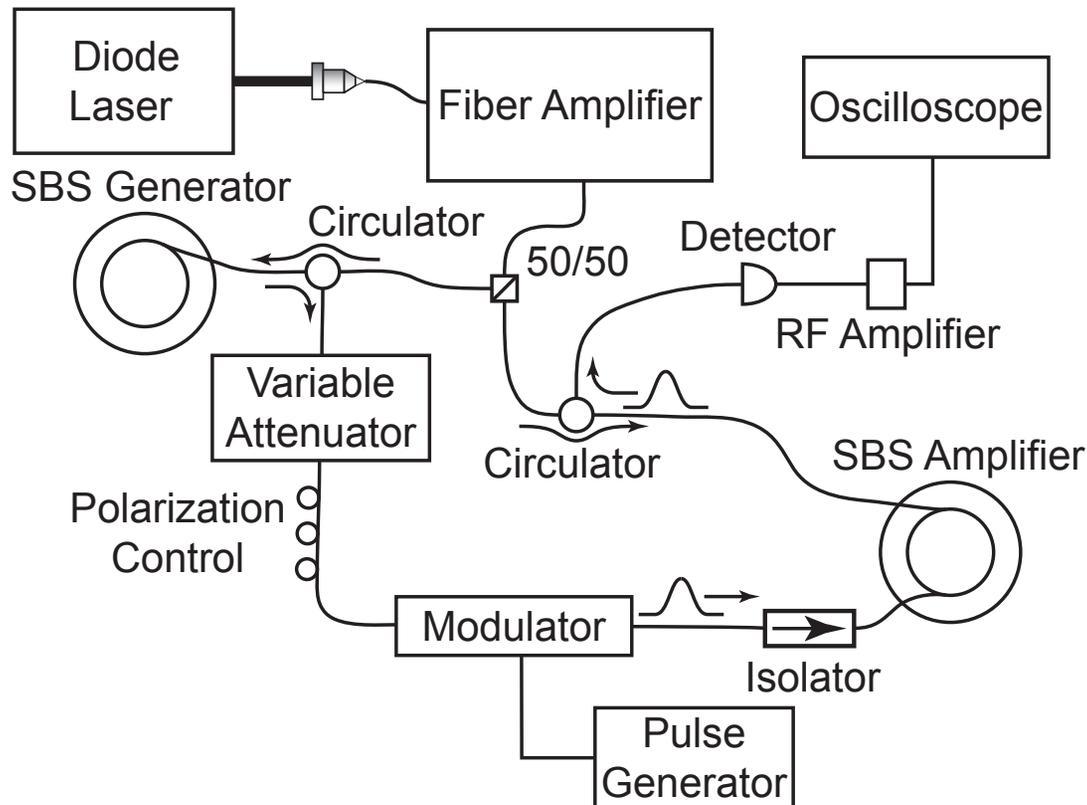
Kam **Wai** Chan

Not to mention

Lene **Hau**

# Slow-Light via Stimulated Brillouin Scattering

- Rapid spectral variation of the refractive response associated with SBS gain leads to slow light propagation
- Supports bandwidth of 100 MHz, large group delays
- Even faster modulation for SRS



# Business was slow at the local Hampton Inn



Thank you for your attention!

And thanks to NSF and DARPA for financial support!

Our results are posted on the web at:

<http://www.optics.rochester.edu/~boyd>

# Business was slow at the local Hampton Inn



# Fundamental Limits on Slow and Fast Light

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Slow Light: There appear to be no fundamental limits on how much one can delay a pulse of light (although there are very serious practical problems).\*

Fast Light: But there do seem to be essentially fundamental limits to how much one can advance a pulse of light.

**Why are the two cases so different?\*\*\***

\* Boyd, Gauthier, Gaeta, and Willner, PRA 2005

\*\*\* We cannot get around this problem simply by invoking causality, first because we are dealing with group velocity (not information velocity), and second because the relevant equations superficially appear to be symmetric between the slow- and fast-light cases.

# Why can one delay (but not advance) a pulse by an arbitrarily large amount?

---

Two crucial differences between slow and fast light

(1) First, note that we cannot use gains greater than approximately  $\exp(32)$  at any frequency to avoid ASE. And we cannot have absorption larger than  $T = \exp(-32)$  at the signal frequency, so signal can be measured. (Of course, the argument does not hinge on the value 32.) When examined quantitatively, these constraints impose a limit of at most several pulse-widths of delay or advancement.

$$\frac{\Delta T}{T} = \frac{1}{2} \sqrt{\alpha L}$$

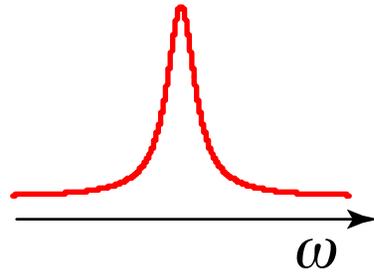
One can overcome these constraints by using a deep hole in an absorption feature, but this trick works only for slow light, as we have just seen.

(2) Spectral reshaping of the pulse is the dominant competing effect in most slow/fast light systems. This also behaves differently for slow and fast-light systems, as we shall now see.

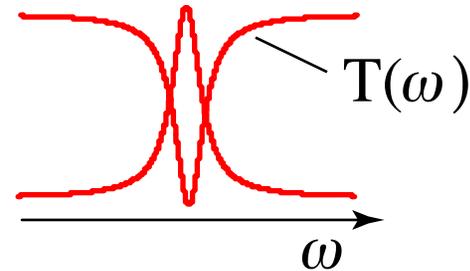
# Influence of Spectral Reshaping (Line-Center Operation, Dip in Gain or Absorption Feature)

---

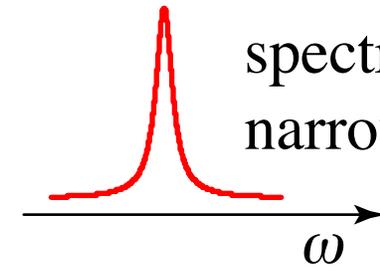
input pulse



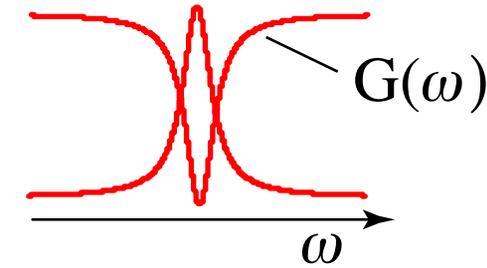
output pulse  
slow-light



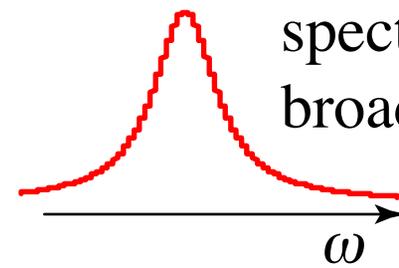
spectrally  
narrowed pulse



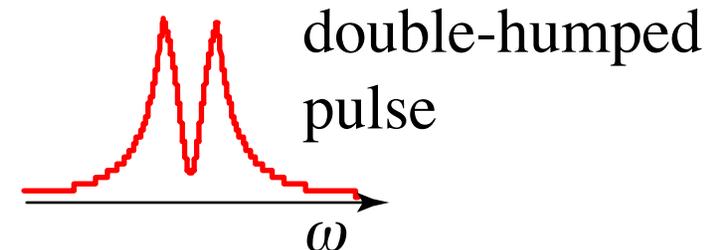
output pulse  
fast-light



spectrally  
broadened pulse



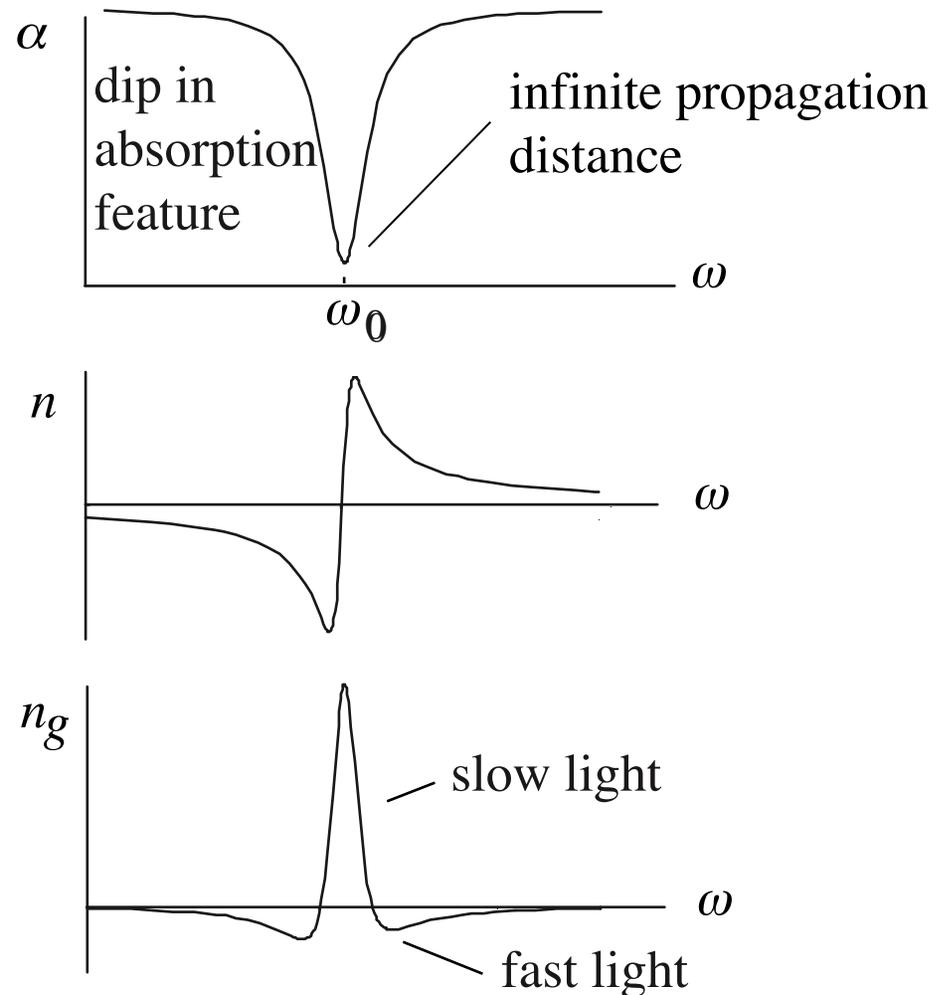
for still longer propagation  
distances, the pulse breaks  
up spectrally and temporally



# Why is there no limit to the amount of pulse delay?

At the bottom of the dip in the absorption, the absorption can in principle be made to vanish. There is then no limit on how long a propagation distance can be used.

This “trick” works only for slow light.

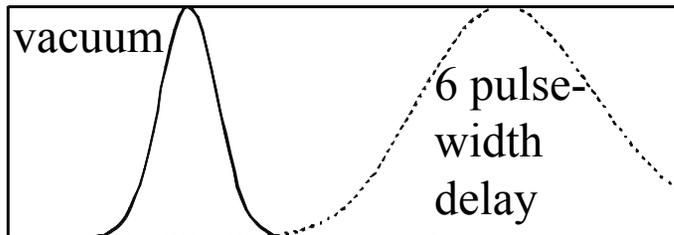
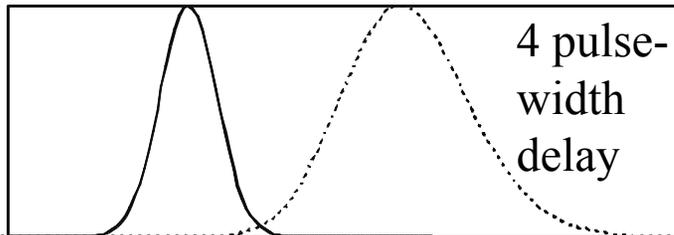
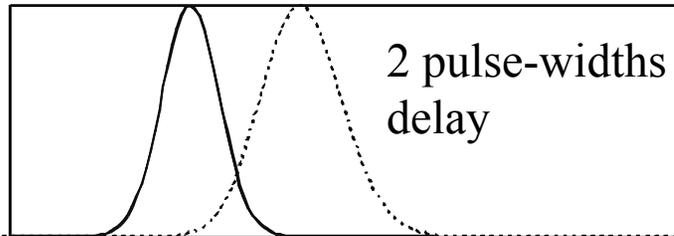
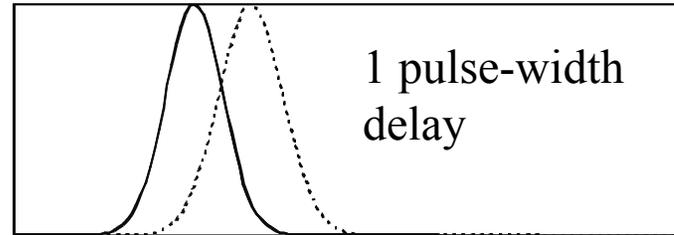


# Numerical Results: Propagation through a Linear Dispersive Medium

Fast light:

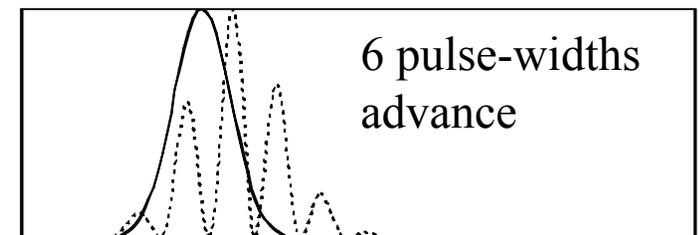
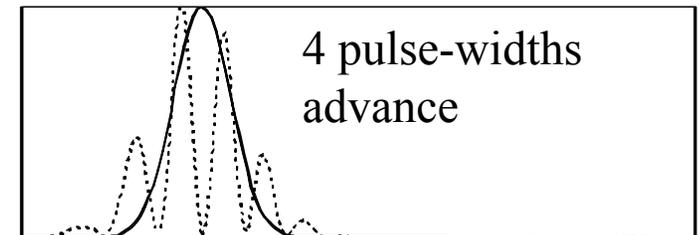
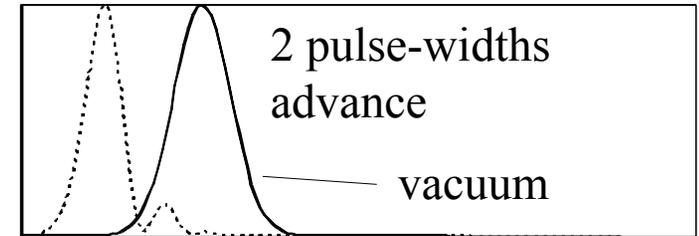
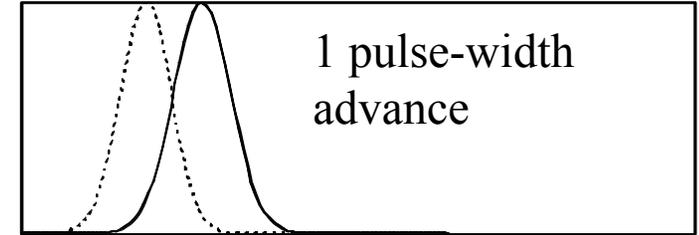
Lorentzian  
absorption line  
 $T = \exp(-32)$   
vary line width  
to control advance

Slow Light



time

Fast Light



time

Slow light:

Lorentzian  
gain line  
 $T = \exp(+32)$   
vary line width to  
control delay

Same Gaussian input  
pulse in all cases