

Phase retrieval as an optical metrology tool

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ABSTRACT

Phase retrieval can be useful in the measurement of optical surfaces and systems. It distinguishes itself through the simplicity of the experimental apparatus, just a detector array which collects light near a focal plane. Aspherics can be measured without null optics. The challenging part of the method is the estimation of the wavefront from the near-focus intensity measurements to reconstruct the wavefront.

Keywords: Phase retrieval, optical metrology, aspheric testing

1. INTRODUCTION

Measuring optical wavefronts precisely is a difficult task because detectors directly sense intensity, not phase. The most common method of measuring the phase uses an interferometer that creates an intensity distribution from which the phase of the field can be determined more easily. Phase retrieval attempts to solve for the complex field using only simple measurements of the intensity of the field. Figure 1 shows a possible experimental arrangement. A known field illuminates and is reflected off of the surface under test. The resulting field propagates through free space to a plane where the resulting intensity pattern is measured using a detector array such as a charge coupled device (CCD). From this measurement the phase retrieval algorithm computes an estimate of the wavefront.

For comparison, Figure 2 shows a single-pass interferometric test, including a reference arm containing a compensating optic. The phase retrieval measurement requires no reference arm optics whatsoever and being more common path it is less sensitive to vibrations. It also avoids the need for a null optic for testing aspherics.

Phase retrieval has been successfully employed in situations where it would be impossible or impractical to use elaborate apparatus to measure the wavefront. In particular it was used to diagnose the aberrations of the Hubble Space Telescope,¹ and will be used to align the segments of the James Webb Space Telescope. However, the technique has not been developed to a point where it is a practical tool for precision optical shop metrology.

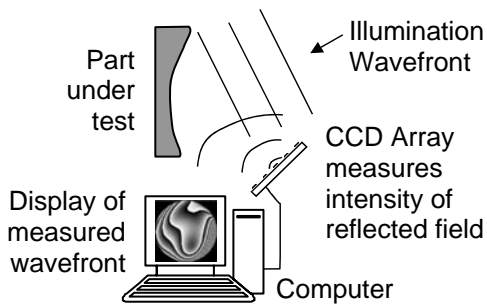


Figure 1: Experimental arrangement for phase retrieval measurement of an optical surface.

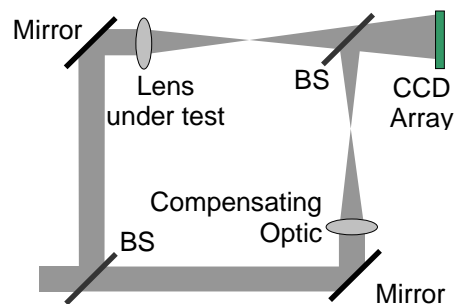


Figure 2: Conventional single pass interferometric arrangement.

2. PHASE RETRIEVAL ALGORITHM

The phase retrieval algorithm used in this work is a gradient search nonlinear optimization algorithm.² It allows us to specify the phase in terms of the coefficients of Zernike polynomials or a point-by-point phase map. It does this by making a starting guess for the field, propagating the guess numerically to the measurement planes, and comparing this to the measurements. If they do not agree, a new guess is formulated and another iteration is performed. We have employed multiple longitudinally separated measurements of the intensity pattern produced by a wavefront as the input to our algorithm, as shown schematically in Figure 3. Propagating sampled fields with a Fresnel transform results in

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sample spacings that are slightly different in each propagated plane. To maintain constant sample spacing for different planes, we first propagate to the paraxial focal plane using the usual Fresnel transform. Then we use the angular spectrum method,³ which maintains sample spacing, to propagate from the paraxial focal plane to each of the different measurement planes.

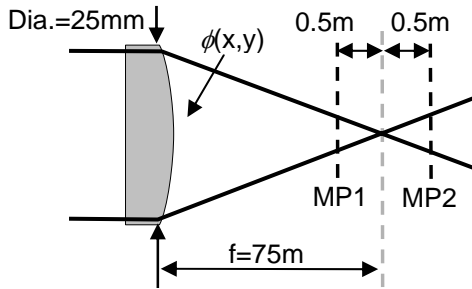


Figure 3: Schematic of modeled experimental arrangement. MP1 and MP2 are the two measurement planes, $\phi(x,y)$ is the phase we are attempting to retrieve.

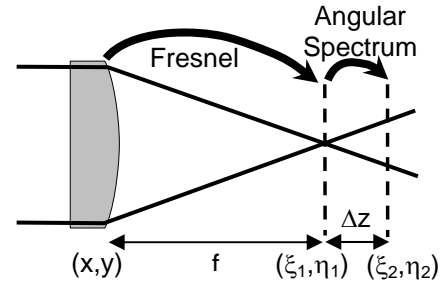


Figure 4: Two step propagation model using Fresnel propagation to the focal plane followed by angular spectrum to a defocused plane.

The metric we have been using is a simple sum of squared errors metric, $E = \sum_{j=1}^J \sum_{r,s} W(r,s) (|F_j(r,s)| - |G_j(r,s)|)^2$

where $|F_j(r,s)|$ is the square root of the measured intensity in the j^{th} plane and $G_j(r,s)$ is the field in the j^{th} plane computed from our assumed aberrated input field. The function $W(r,s)$ allows us to eliminate the contribution of bad pixels. Other error metrics are possible, but this one is very simple to compute and understand.

We are fortunate that simple analytic expressions for the gradient of E can be derived for the parameters over which we wish to optimize.^{2,4} Here we optimize over the coefficients of a set of Zernike polynomials.

3. SIMULATION RESULTS

Several simulations were performed to explore the characteristics of the phase retrieval algorithm. The situation that was modeled is shown in Figure 3. Wavefront aberrations $\phi(x,y)$ were included using 36 Zernike polynomials, where lower order terms were weighted more heavily. The mean RMS of the ensemble of wavefronts used was 0.55 waves. Two measurement planes were employed, each displaced 0.5 mm from the paraxial focus and separated by 1 mm. An f/3 lens was assumed. Care was taken to ensure that the modeled intensities in the measurement planes were at least Nyquist sampled.

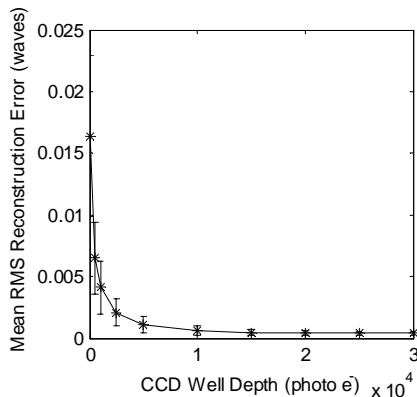


Figure 5: Decreasing well depth increases the reconstruction error. Each point is a mean of 200 reconstructions. Error bars are the standard deviation.

3.1 Reconstructions in the presence of noise

To model phase retrieval realistically, one must consider the effect of noise inherent in making a measurement of intensity. The major noise sources are shot (photon) noise and Gaussian noise due to the readout electronics of the detector array. The SNR due to shot noise is the square root of the number of detected photoelectrons and is limited by the detector pixel well depth. Figure 5 shows the affect of using devices with various well depths on the reconstruction

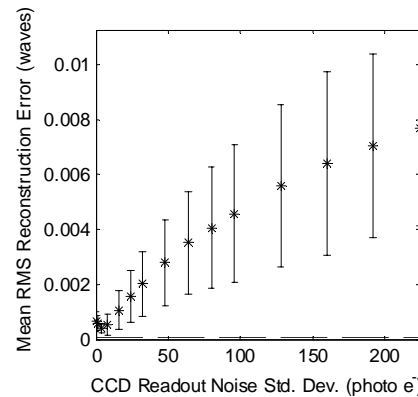


Figure 6: Increasing read noise increases the reconstruction error. Each point is a mean of 200 reconstructions. Error bars are the standard deviation.

error. Our results show errors under $\lambda/200$ (RMS) for the very small well depth of 500 photoelectrons, with a constant read noise of 4 photoelectrons.

The Gaussian read noise is independent of the signal value, so is approximately constant over the pixels in a given frame. Figure 6 shows that increasing read noise increases the error in the reconstructed wavefront. The error is under $\lambda/200$ for read noise values under 100 photoelectrons with a constant well depth of 3×10^4 photoelectrons. Noise parameters in the ranges examined here are achievable with moderately priced CCD cameras. Our results demonstrate phase retrieval's robustness to noise.

3.2 Effect of measurement plane location

The plane of best focus is not the optimum location to measure the intensity patterns used in phase retrieval. This is because of the large dynamic range present in the focal plane, which results in very low SNR's in the many pixels with small values. We have begun investigating this problem by conducting a series of simulations where the separation between the two measurement planes is varied, while keeping the two planes equally spaced about the paraxial focus. These results are shown in Figure 7. Error is a maximum for small plane separations, where the SNR for most of the pixels in the measurement is poor, the dynamic range in the image is large and the two planes do not provide much diversity of measurement. As the planes move away from focus, the error is reduced as light is distributed more evenly in the measurement plane. As the planes move farther apart, the error increases again as the propagation model breaks down due to aliasing in the DFT's. The algorithm "converges" when the error metric is low, meaning that the solution is consistent with the measurements (to within the noise). The algorithm has the most trouble converging when the measurement planes are near focus, and improves markedly when we separate the planes somewhat. When the planes are very widely separated (> 1 mm in this case), convergence is excellent; this is, however, due to aliasing in the propagation; multiple solutions exist for which the error metric is small and the algorithm may not converge to the correct solution.

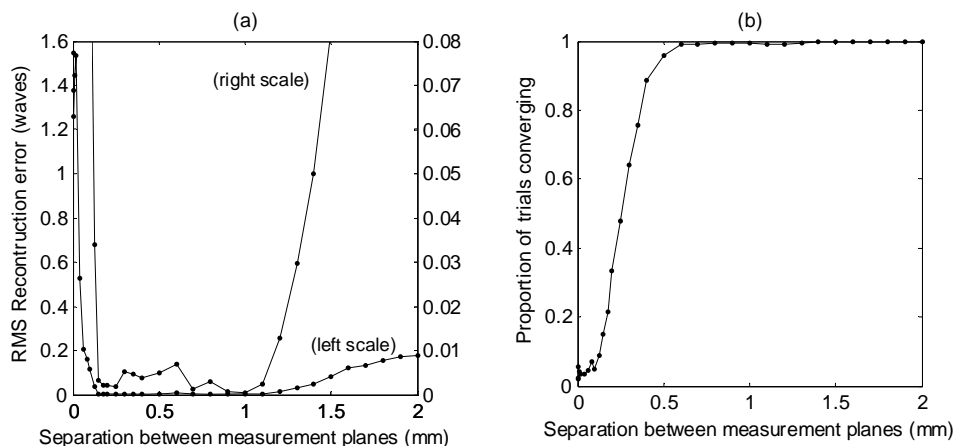


Figure 7: Effect of plane position on (a) reconstruction error and (b) algorithm convergence. The upper curve in (a) is simply the lower curve with the vertical scale magnified 20 times, as indicated by the scale on the right.

4. CONCLUSION

Phase retrieval presents an additional tool for the optical metrologist. It allows measurements of wavefronts with accuracy and range comparable to that of interferometry without the complexity of the interferometric arrangement and associated compensating or null optics. Simulations indicate that the method is robust in the presence of noise.

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