Entanglement and Bell’s Inequalities

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In this experiment we explore the relationships of quantum entanglement by relating photon polarization with parametric down conversion of the Beta Barium Borate (BBO) crystals. We observe $\cos^2$ dependence and violate Bell’s inequality to prove this entanglement.

BACKGROUND

Quantum Entanglement is when a system of objects are related through a property that can not be separated.

$$|\Psi\rangle \neq |1\rangle_s |2\rangle_i \quad (1)$$

One would not be able to adequately express one object without including the other object. In quantum mechanics a specific state does not exist until one measures it. For particles that are entangled if we observe the state of one particle we automatically know what state the other entangled particles have without measuring them. All of these ideas originate after the talk between Bohr and Einstein on the EPR paradox [1]. What is even more puzzling is the fact that this entanglement does not matter how far apart these objects are. This phenomenon is called non-locality, where the distance of the entangled particles is not a factor for this correlation.

THEORY

To prove that photons are entangled through polarization we show two things. The first proof is to show that as we rotate the polarizer of the signal (or idler) and keep the other polarizer of the idler (or signal) fixed we should see a $\cos^2$ dependence in the coincidence count. Second we violate Bell’s inequality using the maximizing angles. A violation would require at least $V \geq 71\%$ and $S > 2$.

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \times 100\% \quad (2)$$

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b') \quad (3)$$

Where $E$ is defined in Eq[4], and $N$ is the coincidence count with $\alpha$ and $\beta$ angles of the polarizers.

$$E = \frac{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) - N(\alpha, \beta_\perp) - N(\alpha_\perp, \beta)}{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) + N(\alpha, \beta_\perp) + N(\alpha_\perp, \beta)} \quad (4)$$

To start off the laser outputs a linearly polarized beam. Right after the Beta Barium Borate (BBO) crystals where the photons entangle through spontaneous down conversion the state can be written as Eq[5]. Spontaneous down conversion exists due to the high non-linear effects of the BBO crystals. The photons go through the BBO crystals with one polarization say $|V\rangle$ (or $|H\rangle$) and wavelength $\lambda$ and after the BBO two photons come out with $|H\rangle$ (or $|V\rangle$) and wavelength $2\lambda$.

$$|\psi\rangle = \frac{|V_\alpha\rangle_s |V_\alpha\rangle_i + |H_\alpha\rangle_s |V_\alpha\rangle_i}{\sqrt{2}} \quad (5)$$

It is also worthwhile to prove that the state is independent of basis. Eq[6] is the states in an arbitrary basis denoted by a arbitrary rotation angle $\alpha$.

$$\left( \begin{array}{c} |V_\alpha\rangle \\ |H_\alpha\rangle \end{array} \right) = \left( \begin{array}{cc} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{array} \right) \left( \begin{array}{c} |V\rangle \\ |H\rangle \end{array} \right) \quad (6)$$

Now we use Eq 6 to put back into Eq 5 to see that the entangled state is independent on the basis.

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|V_\alpha\rangle_s |V_\alpha\rangle_i + |H_\alpha\rangle_s |V_\alpha\rangle_i)$$

$$= \frac{1}{\sqrt{2}} (\cos(\alpha)|V\rangle_s - \sin(\alpha)|H\rangle_s) (\cos(\alpha)|V\rangle_i - \sin(\alpha)|H\rangle_i)$$

$$+ \frac{1}{\sqrt{2}} (\sin(\alpha)|V\rangle_s + \cos(\alpha)|H\rangle_s)(\sin(\alpha)|V\rangle_i + \cos(\alpha)|H\rangle_i))$$

$$= \frac{1}{\sqrt{2}} (\cos^2(\alpha)|V\rangle_s |V\rangle_i + \sin^2(\alpha)|H\rangle_s |H\rangle_i)$$
\[-\frac{1}{\sqrt{2}}(|\psi\rangle_{s}|V\rangle_{i} + |H\rangle_{s}|V\rangle_{i})\]

The photons produced from the down conversion in the BBO crystals we call \(|\psi\rangle\). The probability for the coincidence detection is:

\[P_{VV}(\alpha, \beta) = |\langle V_{\alpha}|(V_{\beta} | \Psi_{DC})|^{2}\] (7)

\[|\Psi_{DC}\rangle = \cos(\theta)|H\rangle_{s}|H\rangle_{i} + e^{i\phi}\sin(\theta)|V\rangle_{s}|V\rangle_{i}\] (8)

Using the the general basis in Eq[6], Eq[7] and the general down conversion state equation Eq 8 which includes the phase difference correction of the Quartz plate, \(\phi\), and original polarization angle from the laser, \(\theta\) we have probability of coincidence below.

\[P_{VV}(\alpha, \beta) = \sin^{2}(\alpha)\sin^{2}(\beta)\cos^{2}(\theta) + \cos^{2}(\alpha)\cos^{2}(\beta)\sin^{2}(\theta) + \frac{1}{4}\sin(2\alpha)\sin(2\beta)\sin(2\theta)\cos(\phi)\left[2\right]\]

Further when \(\theta = \frac{\pi}{4}\) and when \(\phi = 0\) or when \(|\Psi_{DC}\rangle = |\psi\rangle\) we arrive at our observation of which lets us see the \(\cos^{2}\) dependence clearly. From this we deduce that the state is entangled Eq[9].

\[P_{VV}(\alpha, \beta) = \frac{1}{2}\cos^{2}(\beta - \alpha)\] (9)

Lastly we use the maximizing angles of Bell’s inequality we violate it. The angles for the polarizers we are going to use are \((0^\circ, 45^\circ, 90^\circ, 22.5^\circ, 67.5^\circ, 112.5^\circ)\). Arriving with both \(S > 2\), visibility of 71% and a \(\cos^{2}\) dependence in the polarizers will be sufficient to prove entanglement.

**EXPERIMENTAL SETUP**

In this experiment we use:

- Argon Ion laser that has a beam of wavelength, \(\lambda = 363.8\text{nm}\), and is linearly polarized. We use two mirrors to align the laser beam to be parallel to the table. We kept the laser at a power of about 90 mW. Since this is a high powered laser we must use eye protection and note reflection of the laser beam. Also the power up procedure of the laser now includes liquid cooling.

- The laser beam then travels through the BBO crystals where parametric down conversion occurs. The BBO crystals are made up of two parts, both of which makes two photons of \(2\lambda\) when a photon of \(\lambda\) goes through it. For example if a \(|H\rangle\) (or \(|V\rangle\)) goes through we expect to see \(|VV\rangle\) (or \(|HH\rangle\)). So the crystals change polarizations due to its high non-linearity effect. The BBO crystals are identical just perpendicular to each other to entangle both polarizations.

- Next a circular cone is observed as the laser beam travels through the BBO crystals. We adjusted the BBO alignment to get the cones required for observation in Fig 3. We need the light cones to overlap right into our detector.

- The parametric down converted photons are then passed through a interference filter to filter out the not entangled photons. The beam goes through focusing optics and gathered through a detector to record the counts through our Avalanche Photon Diodes. We must also note the time card in our computer that resolved the coincidence counts for us. Our time card has 26ns resolution.

- Lastly we have a beam stopper to stop the remain-
FIG. 3: Using 98mW power Argon Ion laser and adjusting the angle of the BBO crystals gave rise to a circular cone of the entangled photons. Initially of wavelength 727.6 nm.

der of the beam that we do not use. This is a special beam stopper because this is a high powered laser in use.

RESULTS

The first objective was to find the optimal vertical, and horizontal angles for our quartz plate to observe the $\cos^2$ dependence. From figure 2 we can see that Vertical Axis of the quartz plate $\approx 203$ will do the best job in correction. Now fixing the Vertical Axis of the Quartz plate $203^\circ$ and rotate the horizontal to figure out a better setting for the horizontal which we can see to be approximately $-1^\circ$ from figure 5. Through repitition of this procedure through further search we find the optimal angles to be Vertical $204^\circ$ and Horizontal $-1^\circ$ for the quartz plate. We have best correction in the polarization and the phase difference in the quartz plate where the graphs intersect. From figure 6 we show the cosine squared dependence which shows that the photons are related and measuring one of them we can gather information about the other no matter the distance they are separated.

The last proof of photon entanglement is to violate Bell’s inequality. We are in great shape because as in figure 6 we have visibility greater than 71%. We use the following angles in Table I. The values of the table were also corrected by subtracting accidental coincidence. This error is subtracted to increase visibility of the data and takes into account the machine error. Accidental coincidence is calculated by the product of the individual counts multiplied by the machine resolution time divided by the exposure time. Our card resolution time is 26 ns and the exposure time for the table is 10 sec. Using the table data we use equations 3 and 4 to calculate $S = 2.28 > 2$! This violates the Bell Inequality. So this proves the photons out of the BBO crystal are entangled through polarization.

<table>
<thead>
<tr>
<th>$\alpha, \beta$</th>
<th>$b = -22.5^\circ$</th>
<th>$b' = 22.5^\circ$</th>
<th>$b_\perp = 67.5^\circ$</th>
<th>$b'_\perp = 112.5^\circ$</th>
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</thead>
<tbody>
<tr>
<td>$a = -45^\circ$</td>
<td>897.65</td>
<td>192.36</td>
<td>204.13</td>
<td>811.64</td>
</tr>
<tr>
<td>$a' = 0^\circ$</td>
<td>713.97</td>
<td>677.63</td>
<td>220.28</td>
<td>218.92</td>
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<tr>
<td>$a_\perp = 45^\circ$</td>
<td>83.64</td>
<td>629.46</td>
<td>656.61</td>
<td>139.42</td>
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<tr>
<td>$a'_\perp = 90^\circ$</td>
<td>299.07</td>
<td>179.32</td>
<td>562.67</td>
<td>711.85</td>
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</tbody>
</table>

DISCUSSION AND CONCLUSION

In conclusion we have produced entangled photons with an Argon Ion laser and two BBO crystals. We showed this by the $\cos^2$ dependence, and by violation of Bell’s inequality. This is an amazing fact that by measuring one photon we instantaneously can have information of the the other pair entangled photon! This is the puzzling fact that lead scientist of the past to study Quantum
Entanglement. As mentioned in the results acquiring the data proved no small effort, but careful analysis of the optimal correction angles in the quartz plate. This was necessary because Bell’s inequalities has regimes where the quantum system might not violate Bell’s inequality. Likewise if the corrections where not as precise we would not even observe the \( \cos^2 \) dependence as shown in the mathematics above. This fact leads up to further question the nature of the mysterious quantum world to try other types of entanglements such as spin or other useful properties of atoms. This fundamental of non-locality continues to push science and has uncovered lots of applications as listed in the manual [2].

FIG. 5: Quartz plate coincidence count we plot vertical = 203° and vary the horizontal angle to find best correction for the phase change. Green is detector polarizers at 45°, Purple is detector polarizers at 135°, Red is detector polarizers at 0°, Teal is detector polarizers at 90°.

FIG. 6: With the optimal conditions for the quartz plate found we choose one polarizer to remain fixed at (0°, 90°, 45°, and 135°) which the other would vary. Here we demonstrate the \( \cos^2 \) dependence in the coincidence count. Using Eq[2] we calculate the visibility of these to be (74%, 71%, 100%, and 98%).

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[2] Lukishova, Svetlana, Lab 1 Entanglement and Bell’s Inequalities University of Rochester Optics Lab (2008)