LAB 1: ENTANGLEMENT AND BELL’S INEQUALITIES

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Abstract

In this experiment we investigate the phenomenon called entanglement by creating polarization-entangled photons through spontaneous parametric down-conversion and carrying out measurements on these entangled photons. We measure coincidence counts for different polarizations as a function of the rotation of a quartz plate to find the point of best alignment. We show that the coincidence counts follow a \( \cos^2 \) dependence with respect to the polarizers. Finally, we show that Bell’s inequalities are violated for specific angles as we typically expect in case of quantum correlations.

INTRODUCTION AND THEORETICAL BACKGROUND

Quantum entanglement is one of the strangest implications of quantum mechanics. Two particles that are entangled are inherently connected to each other and cannot be described using separate wavefunctions. This implies that a measurement carried on one particle immediately supplies information about a second particle, no matter how far apart these two particles may be. Examples of possible entangled states are polarization-entangled photons or spin-entangled photons.

In this experiment, we create polarization-entangled photons through spontaneous parametric down conversion, which is a non-linear process through which one photon of wavelength \( \lambda \) is converted into two photons of wavelength \( 2\lambda \) with very low efficiency - only one photon gets down-converted out of \( 10^{10} \) incident photons. The down-converted photons are emitted in a cone from the crystal. The down-conversion of a photon passing through a type I BBO crystal is shown in the two following figures.
For a type I crystal cut, the incident horizontally polarized photon of wavelength \( \lambda \) is converted into two photons of wavelength \( 2\lambda \) with vertical polarization as shown in Figure 1(i).

For an incident vertically polarized photon of wavelength \( \lambda \), two horizontally polarized photons of wavelength \( 2\lambda \) will emerge from the crystal as shown in Figure 1(ii).

\[ \text{Figure 1(i)} \quad \text{Figure 1(ii)} \]

**Down-converted photons incident on a Type I BBO crystal**

In our experiment, we use two Type I BBO crystals orthogonally placed together such that for 45° incident polarized light, the same number of photons having horizontally or vertically polarized light will be emitted from the crystals.

\[ \text{Figure 2. Polarization-entangled photons produced} \]

In this way, polarization-entangled photons are produced and can be mathematically represented as follows, where \( \frac{|H\rangle + |V\rangle}{\sqrt{2}} \) is the 45° polarized incident photon.

\[
\frac{|H\rangle + |V\rangle}{\sqrt{2}} \rightarrow |\Psi_{EPR}\rangle = \frac{1}{\sqrt{2}} (|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i)
\]

where \( |\Psi_{EPR}\rangle \neq |A\rangle_s|B\rangle_i \)
As the photons travel through the crystal, a phase difference is created between the horizontally polarized and vertically polarized states.

\[
| \Psi_{\text{SPDC}} \rangle = | V \rangle_s | V \rangle_i + e^{i \theta} | H \rangle_s | H \rangle_i \]

In order to take coincidence counts, this phase shift must be removed. The quartz plate is used to compensate for this phase shift such that it offsets the phase difference introduced by the BBO crystals. This happens when phase \( \phi = 0 \) or \( \pi \).

If we measure the polarizations of the two photons in the \(| H \rangle \) and \(| V \rangle \) basis, there are two possible outcomes: both vertical and both horizontal.

\[
| \Psi_{\text{EPR}} \rangle = \frac{1}{\sqrt{2}} (| V \rangle_s | V \rangle_i + | H \rangle_s | H \rangle_i) \]

We can also measure the polarizations with polarizers rotated by an angle \( \alpha \). We use the rotated polarization basis:

\[
| V_\alpha \rangle = \cos \alpha | V \rangle + \sin \alpha | H \rangle \\
| H_\alpha \rangle = -\sin \alpha | V \rangle + \cos \alpha | H \rangle
\]

We prove that the entangled state EPR does not depend on the basis and is also valid for \(| V_\alpha \rangle \) and \(| H_\alpha \rangle \)

\[
| \tilde{\text{EPR}} \rangle = \frac{1}{\sqrt{2}} (| V \rangle_s | V \rangle_i + | H \rangle_s | H \rangle_i)
\]

\[
= \frac{1}{\sqrt{2}} ((\cos \theta | V \rangle_s + \sin \theta | H \rangle_s)(\cos \theta | V \rangle_i + \sin \theta | H \rangle_i) + (- \sin \theta | V \rangle_s + \cos \theta | H \rangle_s)(- \sin \theta | V \rangle_i + \cos \theta | H \rangle_i))
\]

\[
= \frac{1}{\sqrt{2}} [\cos^2 \theta | V \rangle_s | V \rangle_i + \sin \theta \cos \theta | H \rangle_s | V \rangle_i + \cos \theta \sin \theta | V \rangle_s | H \rangle_i + \sin^2 \theta | H \rangle_s | H \rangle_i]
\]

\[
= \frac{1}{\sqrt{2}} [\cos^2 \theta | V \rangle_s | V \rangle_i + \sin \theta \cos \theta | H \rangle_s | V \rangle_i + \cos \theta \sin \theta | V \rangle_s | H \rangle_i + \sin^2 \theta | H \rangle_s | H \rangle_i]
\]

\[
= \frac{1}{\sqrt{2}} [ | V \rangle_s | V \rangle_i + | H \rangle_s | H \rangle_i ] . \text{ Since } \cos^2 \alpha + \sin^2 \alpha = 1 \text{ and the other terms cancel out.}
The coincidence counts between entangled photons is defined as:

$$N(\alpha, \beta) = A \left( \sin^2 \alpha \sin^2 \beta \cos^2 \theta + \cos^2 \alpha \cos^2 \beta \sin^2 \theta + \frac{1}{4} \sin 2\alpha \sin 2\beta \cos \phi \right) + C$$

where $\alpha$ is the angle of polarizer $\alpha$, $\beta$ is the angle of polarizer $\beta$, $\theta$ is the angle of the incident polarization, $\Phi$ is the phase difference between the horizontally and vertically polarized light and $C$ is a factor added to account for imperfections. The angle $\theta$ is fixed at $45^\circ$ and $\Phi = 0$. Then,

$$N(\alpha, \beta) = \frac{A}{2} \left( \sin^2 \alpha \sin^2 \beta + \cos^2 \alpha \cos^2 \beta + 2 \sin \alpha \cos \alpha \sin \beta \cos \beta \right) + C$$

$$= \frac{A}{2} \left( \cos \alpha \cos \beta + \sin \alpha - \beta \right) + C$$

$$= \frac{A}{2} \cos^2 (\alpha - \beta)$$

Hence the coincidence counts depend on the difference of the angles between the two polarizers. The coincidence counts are used to calculate Bell’s inequality. In this case, Bell’s inequalities define the polarization correlation under measurements at different polarizer angles. A version of Bell’s Inequality developed by Clauser, Horne, Shimony and Holt define the possible values of the inequalities to be $|S| \leq 2$. For values of $|S| > 2$, Bell’s Inequality is violated and a quantum correlation is shown. Bell’s inequality can be calculated as follows:

$$S = |E(a,b) - E(a',b')| + |E(a',b) + E(a',b')|$$

$$E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) - N(\alpha, \beta_\perp) - N(\alpha_\perp, \beta)}{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) + N(\alpha, \beta_\perp) + N(\alpha_\perp, \beta)}$$

The angles used to calculate these inequalities are $a = -45^\circ$, $a^\prime = 0^\circ$, $a_\perp = 45^\circ$, $a^\prime_\perp = 90^\circ$, $b = -22.5^\circ$, $b^\prime = 22.5^\circ$, $b_\perp = 67.5^\circ$, $b^\prime_\perp = 112.5^\circ$. Using these angles, we prove that Bell’s inequality is violated.

**PROCEDURE**

1. The experimental setup consists of a ~100mW Argon ion laser with wavelength $\lambda = 363.8$ nm and a vertical polarization. The light from the laser bounces off two mirrors which are used to ensure that the laser beam is travelling in a straight and level direction.

2. The quartz plate is used to compensate for the phase shift introduced by the present in the laser beam due to the argon plasma tube.
3. A mirror re-directs the beam through a pair of BBO crystals that are orthogonal to each other. Most of the beam goes straight through the crystals and hit the beam stop. The down-converted photons with wavelength $2\lambda=727.6$ nm are emitted in cones.

4. A lens and a camera are used to image these cones of down-converted light as shown in Fig 3.

5. The setup is then changed and the lens and camera are replaced by APDs. The cones of down-converted light are detected by a pair of single-photon avalanche photodiodes (APDs) modules. The APD positions are equidistant from the BBO crystal.

6. A polarizer is located in front of each APD so that we can select the polarization state of the photons reaching the detectors.

7. Coincidence counts are measured for different variables in order to calculate Bell’s Inequality and prove quantum correlations.
ANALYSIS AND RESULTS

The image of the down-converted cone of light obtained from the first experimental setup is shown in Fig 5.

Fig 4. Experimental setup

Fig 5. Down-converted light cone
Both singles and coincidence counts are taken for polarizer $\alpha = 45^\circ$ while varying polarizer angle $\beta$ from $0^\circ$ to $360^\circ$. We see that there is a cosine squared dependence with good visibility as shown in the plot below.

![Coincidence counts v/s $\beta$ angle](image)

For $\alpha = 45^\circ$, we can calculate visibility = $\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$ = 0.92

For $\alpha = 135^\circ$, visibility = 0.93

The quartz plate must be aligned such that it compensates for the birefringence introduced by the BBO crystal. We plot the coincidence counts for $\alpha$ and $\beta$ set at specific angles while rotating the quartz plate around its base plate and its vertical axis. The point at which the coincidence curves all intersect indicate the best orientation for the quartz plate.

We first decide to take measurements of coincidence counts for a fixed vertical axis at $0^\circ$ while rotating the quartz base plate from $200^\circ$ to $230^\circ$. We plot the results for both polarizer angles at $0^\circ$, $45^\circ$, $90^\circ$ and $135^\circ$. The results are shown in Fig 7. From this plot, we see that the ideal base plate angle lies around $202-204^\circ$.

We hence fix the quartz base plate at $203^\circ$ and measure coincidence counts while rotating the vertical axis of the quartz plate. The results obtained are plotted in Fig. 8. This plot seems to show that the coincidence counts intersect best at an angle of $-1^\circ$ on the vertical axis of the quartz plate. Since we are not certain that $203^\circ$ is the best angle for the base plate, we repeat coincidence measurements for a fixed base plate at angle $204^\circ$ while rotating around the vertical axis of the quartz plate. These results are shown in Fig 9. Once again, the best intersection for coincidence counts seems to be for the vertical axis at $-1^\circ$. 
Fig 7. Coincidence counts for different polarizer angles while rotating quartz base plate.

Rotating quartz base plate with vertical axis fixed at 0°

Fig 8. Coincidence counts for different polarizer angles while rotating vertical axis of quartz plate.

Rotating quartz plate about vertical axis with fixed base at 203°
We hence take coincidence measurements for the quartz base plate at 204° and the vertical axis at -1°. The results are shown in Fig 10.

**Fig 10. Coincidence counts for fixed angles of polarizer α = 0°, 45°, 90°, 135° as a function of the rotation of polarizer β from 0° to 360°. Exposure time = 10s. Base quartz plate = 204°, vertical axis of quartz plate = -1° for all measurements**
We can now calculate Bell’s inequalities using the polarizer angles tabulated below.

<table>
<thead>
<tr>
<th>α</th>
<th>β</th>
<th>b= -22.5°</th>
<th>b' = 22.5°</th>
<th>b ⊥ = 67.5°</th>
<th>b' ⊥ = 112.5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>a  = -45°</td>
<td>898</td>
<td>192</td>
<td>204</td>
<td>812</td>
<td></td>
</tr>
<tr>
<td>a' = 0°</td>
<td>714</td>
<td>678</td>
<td>220</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td>a ⊥ = 45°</td>
<td>84</td>
<td>629</td>
<td>657</td>
<td>139</td>
<td></td>
</tr>
<tr>
<td>a ⊥ = 90°</td>
<td>299</td>
<td>179</td>
<td>563</td>
<td>712</td>
<td></td>
</tr>
</tbody>
</table>

\[ S = |E(a, b) - E(a, b')| + |E(a', b) + E(a', b')| \]

\[ E(a, b) = 0.688 \]
\[ E(a, b') = -0.626 \]
\[ E(a', b) = 0.422 \]
\[ E(a', b') = 0.542 \]

Then \[ S = |0.688 + 0.626| + |0.422 + 0.542| = 2.28 \]
Thus \(|S| > 2\), Bell’s Inequality is violated.

**CONCLUSION**

We prove that quantum correlations exist for these angles by showing that Bell’s inequality is violated. The value of \( S \) lies between 2 and \( 2\sqrt{2} \) which means it is a perfectly quantum entangled state. We note that the calculated value of \( S \) should be bigger, but a smaller value is obtained because of a few experimental limitations. The quartz plate is very difficult to align perfectly to compensate for the phase difference introduced by the birefringence of the BBO crystal. We also need a smaller iris aperture so that the span of wavelengths captured is smaller. However, we do obtain decent visibility as can be observed from Fig 10. The visibility is calculated to be above 70% for all the polarizer angles. Our calculations of Bell Inequality also account for accidental coincidence counts using the equation:

\[ N_{accidental} = \frac{S_A S_B \tau}{t} \]

where \( S_A \) and \( S_B \) = singles photon count, \( \tau \) = time resolution of system = 26ns, \( t \) = exposure time = 10s. This helps us to account for errors in the measured coincidence counts.