Entanglement and Bell’s Inequality

Lecture 1: Separability and Parametric Downconversion
Entanglement

• Complicated and Fascinating
• Fundamentally challenges our notion of how the universe works

• Over the course of the next three (four?) lectures and lab exercises, you will:
  – Learn how we produce entanglement
  – Learn why entanglement is special
  – Measure entanglement in the lab
Correlation – Simple Model

- Socks – Red or Blue
- “Random” socks
  \[(R_1 \text{ or } B_1) \text{ and } (R_2 \text{ or } B_2)\]
  \[(R_1 + B_1)(R_2 + B_2)\]
  \[R_1R_2 + R_1B_2 + B_1R_2 + B_1B_2\]
  \[RR + RB + BR + BB\]

- “Separable” – (sock 1)(sock 2)
- No correlation between socks
Correlation – Simple Model

- More rigorously
  \[(pR_1 + qB_1)(pR_2 + qB_2)\]
  \[p^2R_1R_2 + pqR_1B_2 + pqB_1R_2 + q^2B_1B_2\]
  
- if \(p=q=1/2\)
  \[(1/4)(RR + RB + BR + BB)\]

- Equal chance of any combination
  - No correlation, \(\#\text{same} = \#\text{different} = 50\%\)
What if socks come in pairs?

• If sock #1 is red, so is sock #2

\[ R_1 R_2 + B_1 B_2 \]

  – Perfect correlation!
    • # same = 100%
    • # different = 0%

  – Not separable!
    • No way to write this as (sock 1)(sock 2)
Correlation vs. Entanglement

• Correlation ≠ entanglement
  – We don’t talk about “entangled socks,” even though they often come in pairs

• Entanglement requires correlation in “any basis”

• More on this in lecture #2

• First we need a more physical model than socks
Polarization & Photons

- Polarization – direction/axis of the Electric Field
- Photon – smallest quantum of EM energy
  - “wave-like particle” or “wave-particle duality”
  - Particle with wave-like properties
    - Wavelength, polarization, etc.
Polarization

• Vector Quantity
  – Arrow pointing to a spot on a unit circle
  – Whiteboard Exercise

• For this lab, we will only worry about linear polarization

Fig. 1: Various polarizations of light
Malus’ Law

- Etienne-Louis Malus (1775-1812)

\[ I = I_0 \cos^2 \theta \]
How do we get entanglement?

- Parametric Down Conversion – Nonlinear Process
  - Atom “wiggles” at half of the pump frequency
  - Assists in the conversion of one “pump” photon into two down-converted photons (“signal” and “idler”)

\[
k_p = k_s + k_i
\]

\[
n_p \omega_p = 2n_s \omega_s \cos \theta_s
\]

for \( \omega_s = \omega_p / 2 \)

\[
n_p = n_s \cos \theta_s
\]
Birefringence

- Birefringent crystals
  - $n$ depends on polarization
  - “Optic axis” – direction of no birefringence*
  - Light propagating in other directions will be split into two beams according to polarization
  - More whiteboard

*for a uniaxial crystal
Phase Matching

- BBO crystal birefringence lets us “phase match” the down-conversion process

\[ n_p = n_s \cos \theta_s \]

Index of refraction of BBO (negative uniaxial crystal)

- blue solid curve: ordinary polarization
- red/dash-dot curve: full extraordinary polarization
- green/dash curve: extraordinary polarization at the phase matching angle (29°)
Creating Entanglement

• Phase matching will occur for only one polarization

• To get the entangled state we want, we need two crystals rotated by 90°
  – One crystal gives Vertical (V) photons
  – The other gives Horizontal (H) photons
Creating Entanglement

- Each crystal emits down-converted light in a cone
- By combining two crystals, we get two overlapping cones
- Lab activity: Observe down-converted light

Fig. 1. Type I spontaneous parametric downconversion. Photons from a pump beam split into pairs of photons.

Fig. 2. Two-crystal downconversion source.
Laser Safety

• Laser Goggles are not just a fashion statement
  — Always wear them if the laser is on, no exceptions
• Avoid staring into the beam (even with goggles!)
  — When kneeling down to reach plugs, face away from the table
• Avoid sticking body parts in the beam (think sunburn)
• Electrical safety
  — high voltages applied to argon tube
Equipment Safety

• AKA “How to not break stuff”
• APDs and CCD camera are very sensitive
  – NEVER turn the room lights on while these are on
  – Make sure to use enough filters to remove pump while CCD is on
  – If APD count rate gets too high, block inputs and then shut them off!
• Lasers run “a little hot”
  – Never turn it on without water cooling active
  – Let the water run for ~15 minutes after turning off laser