Lab. 1: Entanglement and Violation of the Bell’s Inequality

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Abstract

Aim of this classroom-experiment is to test violation of the Bell’s inequality by using polarization-entangled photons. These photons are produced by spontaneous parametric-down conversion in a pair of Type-I BBO crystals.

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1. Introduction

In their famous paper[1] Einstein, Podolsky and Rosen illustrated the conflict between the principle of locality and the principles of the quantum mechanics. Einstein, a strong believer of the principle of locality, concluded that the formulation of quantum mechanics is incomplete. His conclusion generated an idea among several other scientists that quantum mechanics should be reformulated by including additional variables. Later Bohm [2, 3] constructed a “hidden variable” theory of quantum mechanics. In 1965, Bell [4] (see also, [5]) proposed a class of inequalities which are satisfied if the behavior of a system is governed by the principles of locality, and are violated if it is governed by the principles of quantum mechanics. These inequalities are now known as the Bell’s inequalities. The first experimental evidence of violation of the Bell’s inequality was published in Ref. [6]. Later several other experiments (see, for example, [7–9]) confirming such locality violations were also carried out.

In this classroom-experiment we will test violation of Bell’s inequalities by using polarization-entangled photons. We will use the so-called Clauser-Horne form of the Bell’s inequality and will mainly follow the experimental technique developed by Kwiat et. al. [10] (see also, [11, 12]).

2. Theory and Experimental set-up

According to the usual interpretation of the quantum mechanics, there exists certain two particle states with the property that measurement of a chosen variable of one particle completely determines the outcome of the measurement of the corresponding variable of the second particle. Such a situation may arise when both particles are emitted from a common source in some entangled (non-factorized) quantum state, for example,

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|\phi_1\rangle |\chi_2\rangle - |\chi_1\rangle |\phi_2\rangle). \]  

(1)

As pointed out by Einstein, Podolsky and Rosen [1], measurements on such states may violate the principle of locality, because at the time of measurement, the particles may be so far apart that no influence resulting from one measurement can possibly propagate to the other particle in available time.
In this experiment we generate polarization-entangled photons which are produced in two type-I Beta Barium Borate (BBO) crystals by the process of spontaneous parametric down conversion (see, for example, Ref. [13]).

If a horizontally/vertically polarized photon of wavelength $\lambda$ is incident on a type-I cut BBO crystal, then two vertically/horizontally polarized photons of of wavelength $2\lambda$ emerges from the crystal (see Figs. 1). Now if two type-I BBO crystals are placed back to back and an incident beam which is polarized along a direction making $45^\circ$ angle with the horizontal axis, then polarization-entangled photons are produced (see Fig. 2). For a general angle of polarization $\theta$, of the incident beam, these down-converted photons may be
represented by the quantum mechanical state

$$|\Psi\rangle_{DC} = \cos \theta |H\rangle_s |H\rangle_i + \exp[i\phi] \sin \theta |V\rangle_s |V\rangle_i,$$

(2)

where $\phi$ is the phase difference between the two polarization states introduced due to the fact that a horizontally polarized photon travels a larger distance inside the BBO crystals than a vertically polarized photon before getting down-converted.

![Experimental setup diagram](image)

**FIG. 3:** Outline of the principal elements of the experimental set-up.

The principle of the main experimental setup is explained in Fig. 3. The light from the laser (363.8 nm) is passed through a blue filter to remove any unwanted wavelength. Then it pass through a quartz plate and gets reflected by a mirror before entering a pair of BBO crystals which are mounted back-to-back with their optic axes at right angles with respect to each other. Two avalanche photodiodes (APDs) are mounted on the rails to be on two diametrically opposite points on the down-converted cone, and used to detect parametric down-converted photons. Two interference filters are located in front of each APDs to eliminate the effects of the scattered pump beam and also a beam stop is also used to block the pump beam. The polarization state of photons can be selected by rotating two polarizers placed in front of the two APDs. We denote the angle of the polarizer A in the signal channel by $\alpha$ and that of the polarizer B in the idler channel by $\beta$. 

4
The quartz plate can be rotated along a horizontal and a vertical axis to compensate the phase difference $\phi$. It can be shown that in the ideal position of the quartz-plate ($\phi = 0$) and when the pump beam is polarized in $45^\circ$ angle, the coincidence counts at the two APDs are proportional to the square of the cosine of $(\alpha - \beta)$:

$$N(\alpha, \beta) \propto \cos^2(\alpha - \beta). \quad (3)$$

For this particular set-up, the Bell’s inequalities may be given by

$$|S| < 2, \quad (4)$$

where,

$$S = E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta'), \quad (5a)$$

$$E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_1, \beta_1) - N(\alpha, \beta_1) - N(\alpha_1, \beta)}{N(\alpha, \beta) + N(\alpha_1, \beta_1) + N(\alpha, \beta_1) + N(\alpha_1, \beta)}. \quad (5b)$$

A violation of the Bell’s inequality, i.e, $|S| > 2$ implies that the principle of locality is invalid so far the measurements on these polarization-entangled photons are concerned.

3. Procedure

- **Imaging the down-converted light cone by EM-CCD camera:** An EM-CCD camera is used to detect the photons emerging from the BBO crystal and the images and videos of the down-converted light cone are saved. We used an argon ion laser source which generates light beam of wavelength 363.8 nm.

- **System alignment:** At first the linear polarizer just after the laser source is kept at an angle $45^\circ$. Then the BBO crystal is rotated along the horizontal and vertical axes for the position where the photon counts are maximum. Next, the polarizers A and B are placed and both are set at angle $0^\circ$. Their heights are adjusted for the maximum coincidence counts.

In order to verify that the light cones generated by the two crystals are perfectly superposed on each other, it is necessary to verify that the resultant cone is unpolarized. It is done by rotating one of the polarizers (say, B) and noting that the single-photon counts do not change appreciably.
• **Alignment of the quartz plate:** The quartz plate is rotated first along the horizontal and then along the vertical axis and then the corresponding coincidence counts are measured for different positions of the polarizers A and B, namely, $\alpha = \beta = 0^\circ$, $\alpha = \beta = 45^\circ$, $\alpha = \beta = 90^\circ$ and $\alpha = \beta = 135^\circ$. The results are plotted against the positions of the quartz plate. The angles of orientation of the quartz plate are chosen as the values where the curves meet (or come close). This orientation is kept fixed throughout the experiment.

• **Test of entanglement:** The angle $\alpha$ is kept fixed and the angle $\beta$ is varied from $0^\circ$ to $360^\circ$. The coincidence counts are noted and plotted as function of $(\alpha - \beta)$. This process is done for four choices of $\alpha$.

• **Violation of Bell’s inequality:** The coincidence counts are measured for certain choices of $\alpha$ and $\beta$ and the value of $|S|$ is calculated.

4. Results and Analysis

• **Imaging the down-converted light cone by EM-CCD camera:** The image of the down-converted light cone (see Fig. 4) is taken by the EM-CCD camera for camera gain 255 and exposure time 0.3 sec. The spot inside the cone is due to fluorescent light generated inside the laser cavity.

• **System alignment:** The variation of the single photon counts at the APD B with the angle $\beta$ is shown in Fig. 5. It is clear that the single photon counts does not change appreciably.

• **Alignment of the quartz plate:** Fig. 6(a) shows the plot of coincidence counts against the orientation around the vertical axis of the quartz plate for four positions of the polarizers A and B: $\alpha = \beta = 0^\circ$, $\alpha = \beta = 45^\circ$, $\alpha = \beta = 90^\circ$ and $\alpha = \beta = 135^\circ$. The curves come closest when the angle is $36.5^\circ$. In Fig. 6(b) coincidence counts are plotted against the orientation around the horizontal axis of the quartz plate for three positions of the polarizers A and B: $\alpha = \beta = 0^\circ$, $\alpha = \beta = 45^\circ$ and $\alpha = \beta = 90^\circ$. The curves come closest when the angle is $-0.5^\circ$.

• **Test of entanglement:** Figure 7 shows the dependence of the coincidence counts
on $\beta$. The dependence is almost cosine square as predicted by Eq. (3). Ideally the dependence should be plotted against $(\alpha - \beta)$. However since $\alpha$ is a constant for each curve, the plot against beta shows more or less the same dependence. This cosine square dependence shows that the photons are polarization-entangled.

From Fig. 6 the maximum visibility is found to be 90%.

- **Violation of Bell’s inequality:** The data collected for the calculation to test the Bell’s inequality is given in Table I. From this table it is found that

$$|S| \approx 2.64.$$  \hfill (6)
FIG. 5: Variation of single-photon counts with the angle $\beta$

(a) Alignment of the quartz plate around vertical axis.  
(b) Alignment of the quartz plate around horizontal axis.

FIG. 6: Alignment of the quartz plate.

Clearly the Bell’s inequality ($|S| < 2$) is violated.

6. Conclusions and Discussions

Hence we have tested the violation of the Bell’s inequality by using polarization entangled photons. The most crucial part of this experiment was to align the quartz plate properly.
FIG. 7: Cosine square dependence of the coincidence counts with the angle $\beta$, for $\alpha = 45^\circ$ and $\alpha = 135^\circ$.

6. Acknowledgements

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References and Notes


