Lab 1 : Entangled Photon and Bell’s Inequalities

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In this experiment we create entangled photons by using two BBO crystals and verify various properties of entangled photons. We observe the cosine squared dependence of coincidence on angles of polarization with visibility close to 100% as expected. We test the Bell’s Inequalities and get $S = 2.64 \pm 0.08$ which violates the inequalities as predicted.

1. Background and Theoretical explanation

1.1) Historical Review of Entanglement

In development of Quantum Theory in the early of 20th century, one successful step in physics is discovering Schrödinger equation which can precisely predict the experimental results like no one has ever done before. But one consequence from the equation is pointed out by A. Einstein and his colleagues in their infamous paper [1], which is well-known as EPR paradox, that this result in turn seems to deny locality and inconsistent which his famous theory, General Relativity. This consequence leads to the instantaneous communication between two parts of the system regardless of space-time dependence. In his famous paper, Einstein suggested that quantum theory has not yet completed, there are still some missing piece of jigsaw in the theory and also proposed a theory, so called (global) hidden variable, to fill out the hole in the theory. On the other side, Bohr, who is one of the pioneers in formulating Quantum Theory, suggested that this kind of system cannot be considered as a two system separately since the first place. Consequently, no matter how far they are apart, both particles don’t need any kinds of communication and the Quantum Theory is complete. At that time no one could verify Einstein’s proposition or Bohr’s suggestion until 1964, J.S. Bell published his famous mathematical inequalities which provide us the upper bounded of measurement of any abstract system which behaves locally [2]. This inequalities, in 1969, is re-derived by J. Clauser, M. Horne, A. Shimony, and R. Holt for an optical system [3]. Specifically they derived a new version of Bell’s inequalities for a measurement of coincidence of two different polarization photons. About a decade later, A. Aspect succeeded in carrying out the experiment to verify this inequalities and he found that the inequalities is violated in an experiment with entangled system which implies that entanglement does not behave locally and Einstein’s idea about hidden variable is not correct[4]. Entangled state is a state of a system of particles which cannot be described by the state of each particle individually. In mathematical language, the state space of the system cannot be factorized into the state space of each particle.

$$|\Psi_{12}\rangle \neq |\Psi_1\rangle \otimes |\Psi_2\rangle$$

(1)

Locality in this context refers to there are nothing can travel with speed greater than light and two parts of the system separated in space can communicate to each other with at most at light speed. Having this constraint violated is referred as non-locality.
1.2) Theoretical Explanation

For simplicity, let consider the entanglement of two particles of two-state system and for convenience in the future referring, let the system be the two-photon entanglement in polarization. This system each photon is characterized by its polarization, denote vertically polarized by $|V\rangle$ and horizontally polarized by $|H\rangle$ and use the convention that the bra or ket on the left is the state for photon number one and the bra or ket on the right for the other photon. In this case we can choose

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|V\rangle|V\rangle + |H\rangle|H\rangle)$$  \hspace{1cm} (2)

Notice that this state is not equal to the state obtained by performing the outer product of two state spaces of each particle

$$|\psi\rangle = \frac{1}{2}(|V\rangle|V\rangle + |V\rangle|H\rangle + |H\rangle|V\rangle + |H\rangle|H\rangle)$$ \hspace{1cm} (3)

In CHSH inequalities, the system is considered about measurement of coincidence count from two detectors. The inequalities states that

$$S = |E(a, b) - E(a, b')| + |E(a', b) + E(a', b')|$$ \hspace{1cm} (4)

where

$$E(a, \beta) = \frac{N(a, \beta) + N(a_\perp, \beta_\perp) - N(a_\perp, \beta) - N(a, \beta_\perp)}{N(a, \beta) + N(a_\perp, \beta_\perp) + N(a_\perp, \beta) + N(a, \beta_\perp)}$$ \hspace{1cm} (5)

$\alpha, \beta$ are angles of two polarizers which are placed in front of detectors and the subscribe means the angle which is perpendicular to that angle

$N(a, \beta)$ is a coincidence rate when two polarization is set at angle $\alpha, \beta$

Speak roughly, the quantity $E(\alpha, \beta)$ relates to a probability for measuring two photons at several given polarization. This form, CSHS inequalities, of the Bell’s inequalities will give the upper bounded of the measurement of classical system as 2. A violation of this inequality implies the system does not behave locally. By verifying this violation we can show that entanglement is a system with behave non-locally as suggested by quantum theory and local hidden variable theory is not true. Before we can simplify this expression, we first show one useful property of the entangled state that is the state is independent of the choice of basis using in writing down the state. To show this we first note that changing the basis corresponds to rotating the state, hence
\[
\begin{align*}
|V\rangle &= \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} |\hat{V}\rangle - \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix} |\hat{H}\rangle \\
|\hat{H}\rangle &= \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} |\hat{V}\rangle + \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix} |\hat{H}\rangle
\end{align*}
\]

Therefore (2) becomes
\[
|\psi\rangle = \frac{1}{\sqrt{2}} \left( (\cos \theta |\hat{V}\rangle - \sin \theta |\hat{H}\rangle)(\cos \theta |\hat{V}\rangle - \sin \theta |\hat{H}\rangle) + (\sin \theta |\hat{V}\rangle + \cos \theta |\hat{H}\rangle)(\sin \theta |\hat{V}\rangle + \cos \theta |\hat{H}\rangle) \right)
\]
\[
= \frac{1}{\sqrt{2}} \left( \cos^2 \theta |\hat{V}\rangle|\hat{V}\rangle - \sin \theta \cos \theta |\hat{H}\rangle|\hat{V}\rangle - \sin \theta \cos \theta |\hat{V}\rangle|\hat{H}\rangle + \cos^2 \theta |\hat{H}\rangle|\hat{H}\rangle \\
+ \sin \theta \cos \theta |\hat{H}\rangle|\hat{V}\rangle + \sin \theta \cos \theta |\hat{V}\rangle|\hat{H}\rangle + \sin^2 \theta |\hat{V}\rangle|\hat{V}\rangle \right)
\]
\[
= \frac{1}{\sqrt{2}} \left( |\hat{V}\rangle|\hat{V}\rangle + |\hat{H}\rangle|\hat{H}\rangle \right)
\]

This result allows us to use any two angles which are perpendicular to each other as a basis to represent the entanglement state. Namely, in the expression for \( E(\alpha, \beta) \) which consists of two sets of two angles which are perpendicular to each other. We can call it vertical and horizontal. And any results we get on basis \( |H\rangle, |V\rangle \) are true in general.

With this fact, \( E(\alpha, \beta) \) can be written as
\[
E(\alpha, \beta) = P_{VV}(\alpha, \beta) + P_{HH}(\alpha, \beta) - P_{VH}(\alpha, \beta) - P_{HV}(\alpha, \beta)
\]

where

\( P_{VV} \) is the probability for measuring both photons as vertically polarized photons

\( P_{HH} \) is the probability for measuring both photons as horizontally polarized photons

\( P_{VH}, P_{HV} \) are the probabilities for measuring one photon as vertically polarized photon and the other as horizontally polarized photon. The order of subscribes represent the result from each detector. They can be explicitly written as following
\[
P_{VV} = \frac{N(\alpha, \beta)}{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) + N(\alpha_\perp, \beta) + N(\alpha, \beta_\perp)}
\]
\[
P_{HH} = \frac{N(\alpha_\perp, \beta_\perp)}{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) + N(\alpha_\perp, \beta) + N(\alpha, \beta_\perp)}
\]
\[
P_{HV} = \frac{N(\alpha, \beta)}{N(\alpha, \beta) + N(\alpha, \beta_\perp) + N(\alpha_\perp, \beta) + N(\alpha_\perp, \beta_\perp)}
\]

\[
P_{VH} = \frac{N(\alpha, \beta_\perp)}{N(\alpha, \beta) + N(\alpha, \beta_\perp) + N(\alpha_\perp, \beta) + N(\alpha_\perp, \beta_\perp)}
\]

(9)

In measurement, (9) can be written as

\[
P_{VV} = |\langle V|\langle V|\Pi(a, b) |BBO\rangle|^2
\]

\[
P_{HH} = |\langle H|\langle H|\Pi(a, b) |BBO\rangle|^2
\]

\[
P_{HV} = |\langle H|\langle V|\Pi(a, b) |BBO\rangle|^2
\]

\[
P_{VH} = |\langle V|\langle V|\Pi(a, b) |BBO\rangle|^2
\]

(10)

Where \(\Pi(a, b)\) is an operator which represents the two polarizers in front of the detectors that rotate the state of photons by angle \(a, b\) denote the angle of each polarizer which respects to a vertical direction, \(a\) for the first detector and \(b\) for the second.

\[
\left(\begin{array}{c}
|V\rangle|V\rangle \\
|H\rangle|V\rangle \\
|V\rangle|H\rangle \\
|H\rangle|H\rangle
\end{array}\right)
\]

(11)

\(\Pi(a, b)\) takes the form

\[
\begin{pmatrix}
\cos a \cos b & -\sin a \cos b & -\cos a \sin b & \sin a \sin b \\
\sin a \cos b & \cos a \cos b & -\sin a \sin b & -\cos a \sin b \\
-\sin a \sin b & \cos a \sin b & \cos a \cos b & -\sin a \cos b \\
\sin a \sin b & -\cos a \sin b & \sin a \cos b & \cos a \cos b
\end{pmatrix}
\]

(12)

Therefore for entangled state in (2) takes the form

\[
|BBO\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\]

(13)
(12) and (13) give
\[
\Pi(a, b) |_{BBO} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos a \cos b + \sin a \sin b \\
\sin a \cos b - \cos a \sin b \\
\cos a \sin b + \sin a \cos b
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos(a - b) \\
\sin(a - b) \\
-\sin(a - b)
\end{pmatrix}
\]

Therefore (10) gives
\[
\begin{align*}
P_{VV} &= |\langle V | (V | \Pi(a, b) |_{BBO}) |^2 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos(a - b) \\
\sin(a - b) \\
-\sin(a - b)
\end{pmatrix}^2 = \frac{1}{2} \cos^2(a - b) \\
P_{HH} &= |\langle H | (H | \Pi(a, b) |_{BBO}) |^2 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos(a - b) \\
\sin(a - b) \\
-\sin(a - b)
\end{pmatrix}^2 = \frac{1}{2} \sin^2(a - b) \\
P_{HV} &= |\langle H | (V | \Pi(a, b) |_{BBO}) |^2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos(a - b) \\
\sin(a - b) \\
-\sin(a - b)
\end{pmatrix}^2 = \frac{1}{2} \sin^2(a - b) \\
P_{VH} &= |\langle V | (H | \Pi(a, b) |_{BBO}) |^2 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos(a - b) \\
\sin(a - b) \\
-\sin(a - b)
\end{pmatrix}^2 = \frac{1}{2} \sin^2(a - b)
\end{align*}
\]

And putting (15) into (8) yields
\[
E(a, b) = \frac{1}{2} \cos^2(a - b) + \frac{1}{2} \cos^2(a - b) - \frac{1}{2} \sin^2(a - b) - \frac{1}{2} \sin^2(a - b)
= \cos^2(a - b) - \sin^2(a - b)
= \cos^2(a - b)
\]

Then finally we can simplify (4) as
\[
S = |\cos^2(a - b) - \cos^2(a - b)| + |\cos^2(\hat{a} - b) + \cos^2(\hat{a} - b)|
\]

This function will give the maximum value 2.82 at the following set of angles \( a = 135^\circ, \hat{a} = 0^\circ, b = 157.5^\circ, b = 22.5^\circ \).
Apart from this another interesting prediction is the first equation in (15), namely

$$P_{vv} = \frac{1}{2} \cos^2(a - b)$$  \hfill (18)

This result shows that the coincidence rate of entangled photons state is cosine squared dependent of the difference of the angles of two polarizers and the curve will have 100% visibility. The visibility is given by

$$Vis = \frac{l_{max} - l_{min}}{l_{max} + l_{min}} \times 100\%$$

where $l_{max}$ and $l_{min}$ are the maximum and minimum coincidence achieved by varying the angle of polarizers respectively.

Plug these in the theoretical maximum and minimum values from (18) we get the prediction

$$Vis = \frac{1 - 0}{1 + 0} \times 100\% = 100\%$$

(20)

Furthermore if we are interested in the count rates from single detector regardless of polarization of the other photon, we see that

$$P_{\text{SINGLE}} = P_{vv} + P_{vh} = P_{hh} + P_{hv}$$

$$= \frac{1}{2} \cos^2(a - b) + \frac{1}{2} \sin^2(a - b)$$

$$= \frac{1}{2} (\cos^2(a - b) + \sin^2(a - b))$$

$$= \frac{1}{2}$$

(21)

That is the single count rates from detectors are constant.

By verifying the violation of Bell’s Inequalities we can prove the presence of entangled photons in our system and also these two predictions, cosine squared dependence and 100% visibility can be observed easily.
2. **Set up, Experiments, and Results**

2.1) **Creating Entangled Photons** [5]

Two BBO crystals type 1 are used as an entangled photons light source. We pump them with Ar-ion laser of wavelength 363.8 nm with definite polarization at 100 mW to produce down-converted photons of wave length 727.6 nm via parametric down-conversion process, SPDC. SPDC is a process which, in short, the crystal absorbs one photon to the excited state. With a small probability the crystal then spontaneously emits the photons by two successive jumping down to the intermediate level and then to the ground level, Fig 1. This process will create two lower energy photons. By cutting the crystal at an appropriate angle we can generate two photons of equal wavelength. The process creates two photons of the polarization perpendicular to the pumping photon, namely pumping with horizontal polarized photon will get two vertically polarized photons, and vice versa. Simplified mathematical expression of this process is

\[ |H\rangle \rightarrow |V\rangle|V\rangle \]

Or

\[ |V\rangle \rightarrow |H\rangle|H\rangle \]

To create entangled photons we need to put the two crystals so that their optical axes are perpendicular to each other and align the pump beam so that its polarization makes an angle of 45 degrees to the optical axes, Fig 2. This way the pumping photons have 50 percents chance to be converted by the first crystal and 50 percents by the second. This is not completely fulfilled the requirement for entangled photons yet. Since the photons produced by the second crystal will have phase shift with respect to the phase of the photons converted by the first crystal, namely

\[ |45\rangle = \frac{1}{\sqrt{2}} (|V\rangle|V\rangle + e^{i\varphi}|H\rangle|H\rangle) \]

To compensate this factor we put the waveplate in front of the crystal to delay the phase of one component of two polarizations so that the down-converted photons will have zero phase shifts. With all
of these procedures, we will obtain the entangled photons as we want. The entangled photons will emerge out off the crystals making light a light cone due to momentum conservation, Fig 3a

![Figure 3: Down-converted light cones. Red ray shows vertically polarized and green ray shows horizontally polarized. The color is not reflected to the true wavelength of photons](image)

Apart from the desired entangled photon, we also have noise from the crystals which consists of two photons from, improperly, down-conversion with different wavelength mixing in the light in the cones. This noise can be removed by inserting two interference filters of band pass 727.6 nm with small bandwidth in front the detector. We also put, for safety, a beam stop to block the unconverted photons which also present after the crystals.

2.2) Detectors

We use two different photon detection systems in the experiment.

2.2.1) APDs and coincidence module

APDs are connected to the coincident count module to measure coincidence. The module is a computer board card connected directly to the computer. The signals from APDs are received by the module and number of photons detected in the experiment and coincidence among these photons will be reported. The time-window of the coincidence of this module is 26 ns, this time relates to the maximum separation in time of two photons detected by detectors that to be regarded as coincidence. This detecting system allows us to set the acquisition time and provides us number of photons found by each detector (single count) and coincidence count. This coincidence counting module will also give us the accidental coincidence count which can be estimated by the formula

\[
\text{AccidentalCoincidence} = \frac{\text{SingleCount}_A \times \text{SingleCount}_B \times \text{TimeWindow}(26 \text{ ns})}{\text{AcquisitionTime}}
\]

2.2.2) CCD camera

This module consists of numerous CCD cells on the detection region of the detection. Each of them has its own amplifier and works independently. The detector provides us the number of photons
found by each cell in the adjustable period of time. We can also construct the image of incident image using the specific program.

2.3) Experiment

We first verify the cosine squared dependence of the coincidence, see equation (18). APDs are used as a detection system in this part. On the other side of the optical table from BBO crystals, two APDs are placed to detect a photon from BBO’s. By inserting two polarizers in front of each detector, we can measure coincidence rate of various combination of two polarization of each photons pair. Also the filters are inserted in front of them to reduce noise from improper down-conversion and background, Fig 4. For simplicity let called the detector on the right hand side as detector A and the left hand side detector as detector B. Also let call the polarizer in front detector A as polarizer A, and polarizer B for the other.

![Figure 4: Demonstrate all main parts of the set up.](image)

First we set the polarizer A to be 45 degrees and the polarizer B to be 0 degrees. Before turning the laser on, measure background noise and dark count of each detector. We found zero background coincidence. Set acquisition time at 5 seconds. Record the following data 1) Single count from detector A. 2) Count rate of detector B. 3) Coincidence. Repeat it three times and find their average. Then change the polarizer B to 10 degrees and measure those quantities again. Repeat this step for every 10 degrees increment of the angle of polarizer B until we rotate it by 360 degrees. Then rotate the polarizer A to 135 degrees and repeat the previous step again. We then subtract accidental coincidence from measured coincidence using equation (23). By plotting the net coincidence vs. angle of polarizer B, cosine squared dependences are clearly seen, Fig 5. Also another interesting result is the count rate from each detector which are shown in Fig 6. Notice that the count rates are fairly constant with visibility about 17% which agrees with (21). And finally we calculate the visibility of coincidence each set to be at 98.2% for polarizer A at 45 degrees and 99.5% for polarizer A at 135 degrees which are close to 100% as expected.
Figure 5: Plots of coincidence as a function of angle of polarizer B

Figure 6: Plots of Count Rates from each Detectors as a function of angle of polarizer B
We then check the violation of Bell’s Inequalities. We measure single counts and coincidence for 20 second for three times when the polarizer A is at 0 degrees and polarizer B is at 22.5 degrees. We then repeat this procedure for polarizer B at 67.5, 112.5, and 157.5 degrees. We then change polarizer A to 45, 90, and 135 degrees, each time measure coincidence when polarizer B is at one of the four angles. We should end up with 16 pairs of angles. Subtract the accidental coincidence, equation (23), and find the average and standard deviation of coincidence from each pair of angles. The result is shown in Table 1.

Table 1 : Coincidence in Verifying Bell’s Inequalities

| Angle of Polari- | Angle of Polari- | 1st Trial | 1st Trial | 2nd Trial | 2nd Trial | 3rd Trial | 3rd Trial | Average (SD) |
|----------------|----------------|-------------|-------------|------------|-------------|-------------|-------------|------------|------------|-------------|------------|-------------|------------|-------------|
| 0              | 22.5           | 88255       | 97272       | 253        | 11 242     | 88255       | 97272       | 256        | 11 245     | 88255       | 97272      | 222         | 11 211     | 244         | 233 (19) |
| 0              | 67.5           | 88670       | 98447       | 91         | 11 80      | 88670       | 98447       | 103        | 11 92      | 88670       | 98447      | 94          | 11 83      | 96          | 85 (6)   |
| 0              | 112.5          | 93516       | 97795       | 165        | 12 153     | 93516       | 97795       | 151        | 12 139     | 93516       | 97795      | 146         | 12 148     | 98         | 130 (10) |
| 0              | 157.5          | 84740       | 89381       | 296        | 10 286     | 84740       | 89381       | 242        | 10 232     | 84740       | 89381      | 267         | 13 228     | 268         | 249 (27) |
| 45             | 22.5           | 81171       | 74592       | 279        | 8 271      | 81171       | 74592       | 286        | 8 278      | 81171       | 74592      | 259         | 13 228     | 275         | 259 (14) |
| 45             | 67.5           | 77660       | 73401       | 234        | 7 227      | 77660       | 73401       | 221        | 7 214      | 77660       | 73401      | 238         | 13 208     | 231         | 216 (9)  |
| 45             | 112.5          | 88844       | 77887       | 51         | 9 42       | 88844       | 77887       | 39         | 9 30       | 88844       | 77887      | 83          | 13 47      | 48          | 40 (23)  |
| 45             | 157.5          | 88053       | 76084       | 71         | 9 62       | 88053       | 76084       | 66         | 9 57       | 88053       | 76084      | 35          | 13 50      | 40          | 20 (35)  |
| 90             | 22.5           | 90654       | 96811       | 140        | 11 129     | 90654       | 96811       | 122        | 11 111     | 90654       | 96811      | 138         | 13 46      | 92          | 133 (10) |
| 90             | 67.5           | 90148       | 93100       | 267        | 11 256     | 90148       | 93100       | 271        | 11 260     | 90148       | 93100      | 239         | 13 195     | 239         | 237 (17) |
| 90             | 112.5          | 90094       | 89758       | 271        | 11 260     | 90094       | 89758       | 243        | 11 232     | 90094       | 89758      | 251         | 13 209     | 255         | 234 (14) |
| 90             | 157.5          | 92459       | 92044       | 117        | 11 106     | 92459       | 92044       | 142        | 11 131     | 92459       | 92044      | 140         | 13 104     | 136         | 114 (17) |
| 135            | 22.5           | 96572       | 117723      | 48         | 13 53      | 96572       | 117723      | 54         | 13 45      | 96572       | 117723     | 56          | 13 59      | 73          | 23 (4)   |
| 135            | 67.5           | 97563       | 118603      | 161        | 15 148     | 97563       | 118603      | 157        | 15 142     | 97563       | 118603     | 145         | 13 60      | 154         | 124 (8)  |
| 135            | 157.5          | 92178       | 108582      | 408        | 13 399     | 92178       | 108582      | 378        | 13 365     | 92178       | 108582     | 391         | 11 329     | 392         | 366 (15) |
| 135            | 112.5          | 92178       | 108582      | 408        | 13 399     | 92178       | 108582      | 378        | 13 365     | 92178       | 108582     | 391         | 11 329     | 392         | 366 (15) |
| 135            | 157.5          | 85934       | 100883      | 313        | 11 302     | 85934       | 100883      | 351        | 11 340     | 85934       | 100883     | 335         | 13 290     | 333         | 310 (19) |

From these data we calculate the quantities quantity $S$ using equation (8) and (9). We obtain $S = 1.98 \pm 0.17$. This result does not completely violate the inequalities, nevertheless the uncertainty of the quantity on plus side is greater that two. This suggests that some devices in the set up are not aligned properly. We narrow all the possibilities down to three choices. 1.) The BBO crystals 2.) The position of the detector. and 3.) The waveplate used in compensating the phase shift

We first investigate the BBO crystals and find out that they are slightly misaligned. After realigning the crystals, we observed the down-converted photons light cones, Fig 3, by placing CCD camera behind the crystals. Lens is used to scale the image so that it is small enough for the detection region of the CCD camera. We can capture the image of down-converted light cone as expected, Fig 7.

After that we check the positions of two detectors so that both of them detecting the photons from the opposite sides of the down-converted light cone. This part is done with help from Dr. Lukishova.

Next experiment is to optimize the phase shift between two two-photon states, equation (22), to achieve a better entangled photons state which will yield better results. Optimizing is done by adjusting the angles of waveplate both vertical axis and horizontal axis. We search for the case when the measured coincidence is independent of the angle of polarizers, equation (7). We carry out this experiment by moving back to use APDs detectors and two polarizers. We measure coincidence from the following pairs of angles of polarizers, 0-0, 45-45, 90-90, and 135-135 with acquisition time of 5 second. We first optimize vertical angle in the range of 50 degrees with 2 degrees increment and fix horizontal angle at 359 degrees. We obtain the optimized angle at 36 degrees, fig 7. We then move to optimize the horizontal angle by fixing vertical angle at 36 degrees and we get the optimized angle at 359 degrees, fig 8. For more precise result, we start the optimizing again with narrower range and finer increment, 6 degrees range for
vertical and 10 degrees for horizontal with increment of 1 degree. The optimized angles we get are 36.5 degrees for vertical angle and 359.5 degrees for horizontal, fig 9 and fig 10.

Figure 7: The image of down-converted light is

Figure 8: Optimizing Vertical Angle
Figure 9: Optimizing Horizontal Angle

Figure 10: Finer Optimizing Vertical Angle
Next we verify the visibility approximately by measuring coincidence for 5 seconds when both polarizers are at the same angles, which is theoretically the maximum, equation (18). Then rotate one of them by 90 degrees and measure the coincidence which is now the minimum theoretically, again by equation (18). We pick the following angles 0, 45, 16, and 100. The results for visibility are 81%, 96%, 88% and 81%, respectively, as shown in Table 2. Obviously, they are all close to 100% in an acceptable range.

Table 2: Approximate Visibility After Optimizing Waveplate

<table>
<thead>
<tr>
<th>Angles (Degrees)</th>
<th>Maximum Coincidence (Count)</th>
<th>Minimum Coincidence (Count)</th>
<th>Visibility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 90</td>
<td>555.00</td>
<td>59</td>
<td>81</td>
</tr>
<tr>
<td>45, 135</td>
<td>611</td>
<td>13</td>
<td>96</td>
</tr>
<tr>
<td>16, 106</td>
<td>530</td>
<td>33</td>
<td>88</td>
</tr>
<tr>
<td>100, 190</td>
<td>507.00</td>
<td>53</td>
<td>81</td>
</tr>
</tbody>
</table>

Finally we check the violation of Bell’s inequalities once again after optimizing all of the devices by carrying out the measurement coincidence for 16 pairs of angles of polarizers. The data obtained this time is shown in Table 3.
Then we calculate quantity $A$ again using equation (8) and (9). We now obtain $S = 2.64 \pm 0.08$ which obviously violates the inequalities. This result implies that we really achieve the entangled photons as we expect.

3. Conclusion

We obtain $S = 2.64 \pm 0.08$ which is greater than 2 and violates Bell’s Inequalities. This result strongly confirms that we have produced entangled photons by using SPDC process in BBO type 1 crystals. Furthermore the cosine squared dependence and visibility about 100% are also confirmed.

4. References

[6] Lukishova, Svetlana, Lab 1 Entanglement and Bell's Inequalities, University of Rochester, Optics Lab (2008)