Quantum Entanglement
and Bell’s Inequalities

Zachary Evans, Joel Howard, Jahnavi Iyer, Ava Dong, and Maggie Han

Institute of Optics, University of Rochester

Opt 101 Meeting, December 4, 2012, Rochester NY
Entanglement, What is it?

- A state of being of two or more particles with special strong correlations
- Allows for reliable conclusions to be made about the state of one by the measurement of the state of the other
- Non local
- Multiple forms of entanglement (Energy, momentum, polarization, spin, etc…)

\[ \left| \Psi_{12} \right> \neq \left| \Psi_1 \right> \otimes \left| \Psi_2 \right> \]
Entanglement, What is it?

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Distance
EPR and Bell

- EPR introduced entanglement, in 1935, but did not believe in it ("spooky action at a distance")
- Einstein disagreed with non-locality, and sought an alternate explanation involving hidden variables to complete quantum mechanical theory.
- In 1964, John Bell developed a series of inequalities which allowed experimentalists to verify entanglement.
- Clauser, Horne, Shimony, and Holt created the commonly-used version of Bell’s Inequality.
- This experiment was made by Freedman and Clauser in 1972, and a more modern version was performed by Aspect in 1981 and 1982.
Experiment:

Set Up

1. Laser
2. Quartz plate
3. BBO Crystals
4. Polarizers
5. Interference Filters
6. Avalanche Photodiode Modules (APD)
Experiment: Set Up

**BE VERY CAREFUL**

Argon Ion Laser
~363.8 nm

Photo Detectors, Collecting System, Polarizers and Interference Filters

BBO Crystals
~727.62 nm
Experiment: BBO Crystals

- Creates Two Cones of Entangled Photons Via SPDC
- $10^{-10}$ Probability of photons SPDC
- Two Vertically Polarized, Two Horizontally Polarized from 45 degree incident polarization

\[
\Psi_{SPDC} = |V \rangle_s |V \rangle_i + e^{i\Phi} |H \rangle_s |H \rangle_i
\]
Experiment: Quartz Plate

- Compensates for the phase difference between the different polarizations that emerge from the BBO Crystal
- Important to have overlapping cones
Collecting the Correct Photons

- Polarizer
  - Selects polarization

- Interference Filter
  - Rejects Laser light

- Microscope Objective
  - Focuses light into the optical fiber
Counting the Photons

Avalanche Photo Detectors
- Each detector detects single photons
- Creates TTL pulses for the computer to read

Computer chip counts the number of electrical pulses from each detector (singles) and simultaneous pulses (coincidences)
Basic Procedure

1. Create SPDC photons in BBO crystals

2. Change relative polarizer angle between polarizer A and B (angle A – angle B)

3. Measure number of simultaneous counts (coincidence count) for that relative angle, and repeat
A series of classical relationships determines whether or not we have achieved entanglement. 

If Bells inequality is violated for some value of parameters then entanglement is shown to occur.

16 Coincidence Count measurements to enter into the inequality and prove entanglement occurred.
How to Prove it

• Bells Inequality's!!

16 Measurements at definite angles alpha and beta

\[
E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) - N(\alpha, \beta_\perp) - N(\alpha_\perp, \beta)}{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) + N(\alpha, \beta_\perp) + N(\alpha_\perp, \beta)}
\]

\[
|a + b + c + ...| \leq |a| + |b| + |c| + ...
\]

\[
S = |E(a, b) - E(a, b')| + |E(a', b) + E(a', b')|
\]

If S is greater than 2, entanglement has be shown to occur
## Results:

### Bell’s Inequality Violation

<table>
<thead>
<tr>
<th>Angles</th>
<th>N Values</th>
<th>E Values</th>
<th>S Value</th>
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<tr>
<td>Polarizer</td>
<td>Polarizer</td>
<td>Coincidence counts</td>
<td>E(a,b)</td>
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<tr>
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</table>

\[
E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) - N(\alpha, \beta_{\perp}) - N(\alpha_{\perp}, \beta)}{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) + N(\alpha, \beta_{\perp}) + N(\alpha_{\perp}, \beta)}
\]

\[
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\]

1 second acquisition time
Fringe Visibility

Dependence of Coincidence Count of Relative Polarizer Angles

Fringe Visibility: $1 > 0.71$
Singles Count

Singles Count Vs. Angle for 90 Degrees

Singles Count vs. Angle for 0 Degrees
Applications Of Entanglement

- Quantum Computing
- Quantum Encryption
Quantum Computing

• Advantages of Quantum Computing
  • Speed up computation, and more powerful computation because the quantum computer might be able to do multiple calculations simultaneously. And it also means parallel calculation because of entanglement
Entanglement in Quantum Cryptography
Ekert Protocol

Alice and Bob each receive one of a pair of entangled photons

Measurements along parallel axes- key generation

Oblique angles- test inequalities

Evesdropping will destroy the entanglement and reduce the degree of violation in Bell's Inequalities.
Thank you

Questions?