Single and Entangled Photon Sources

James MacNeil and Madhu Ashok
University of Rochester
The Institute of Optics
Submitted to Dr. Svetlana Lukishova on 10/21/2013

Abstract: This paper explores the fundamentals and applications of single and entangled photon sources. These sources are the ground works for advances in quantum communications, ranging from quantum cryptography to topics as peculiar as quantum teleportation.
Single Photon Sources

I. What is a Single Photon Source?

A single photon source is an emitter of light that has all of its photons separated in time. It is important to distinguish between an attenuated light source and a single photon source—laser light may be attenuated down to a level such that a single photon is found per meter, but laser light consists of coherent light, which obeys classical statistics. Because of this, attenuated laser light will be contaminated with pairs or triplets of photons. Attenuated lasers are a good approximation of a single photon source, but they are by no means a real single photon source.

What is meant by a real single photon source? As mentioned above, a real single photon source has all of its photons separated in time, that is, it emits antibunched light [1]. This means that when it emits light, it emits a single quanta of light at a time (i.e. single photons all separated in time). Many photons are produced at the same time in lasers. This is why even after attenuation a laser cannot be considered a single photon source—photons that pass through the source of attenuation may have been produced simultaneously and are thus correlated (bunched).

II. Measurement of Single Photon Sources

To quantify the degree of antibunching for a particular source of photons, the second order coherence function is used [1]:

\[ g^{(2)}(r_1, t; r_2, t + \tau) = \frac{\langle I(r_1, t)I(r_2, t + \tau) \rangle}{\langle I(r_1, t) \rangle \langle I(r_2, t + \tau) \rangle} \]

Where \( I \) is the intensity of the light, \( r_1 \) and \( r_2 \) are the positions of two separate detectors, and \( \tau \) is the time between detection of photons. This function is a measure of the joint photo-detection probability for the two detectors at different times [1]. For classical light, \( g^{(2)}(r_1, t; r_2, t + \tau) \leq g^{(2)}(r_1, t; r_2, t) \), meaning that it is more probable for the detectors to register photons at the same time. For antibunched light, \( g^{(2)}(r_1, t; r_2, t) = 0 \), meaning that there is zero probability that two photons will be detected at the same time.

To measure the coherence between photons, a Hanbury-Brown Twiss setup can be used, shown below in figure 1. Fluorescent light enters the setup.

![Figure 1: Hanbury-Brown Twiss setup [1]](image-url)
and hits the beam splitter, which leads to two separate single photon detectors (avalanche photodiodes are shown in the figure). When a photon is incident on one detector, the computer begins measuring time until a photon is incident on the other detector—in this way, the computer measures the time between incident photons. The software compiles these time measurements into a histogram, shown below in figure 2. A histogram compares the number of coincidence counts to inter-photon times. A dip at $t=0$ on the histogram curve corresponds to zero photons detected at the same time and thus antibunched light.

III. Types of Single Photon Sources

Quantum mechanics states that the energy levels of all systems are quantized. On a small enough scale, this discreet quantization of energy becomes noticeable, whereas on larger scales, energy appears to be continuous. Single photons arise from this quantization, where the energy of the emitted photon, $h\nu$, is equal to the spacing between energy levels of a system:

$$\Delta E = h\nu$$

All atoms and molecules are single photon sources. When one of their electrons absorbs a single quanta of energy, it gains kinetic energy and moves further away from the nucleus. When the electron releases energy and returns to its ground state, it releases a photon with frequency corresponding to the decrease in its kinetic energy.

Nitrogen Vacancy Nanodiamond

There are a number of other types of single photon sources—a single photon source can be any system of charged particles with a discreet energy level structure. With regards to implementation, single atoms and molecules are impractical due to their
minute size and inability to be isolated. A different type of single photon source that is more practical is the nitrogen vacancy color center in nanodiamond.

Nitrogen vacancy color centers, shown in figure 3, occur when a lattice vacancy becomes trapped next to a nitrogen impurity in diamonds. Vacancies can be created through electron irradiation at energies greater than 200 keV [2]. The vacancies become mobile at temperatures above 800 degrees Celsius, and can become trapped next to a nitrogen impurity [2]. A portion of the nitrogen’s valence electrons along with electrons in the vacancy form a discrete energy level structure, leading to the emission of single photons with a strong zero phonon line of 1.945 eV [2].

![Figure 3: Nitrogen vacancy color center in diamond lattice. Image taken from Wikipedia page on nitrogen-vacancy nanodiamonds](image)

Nitrogen vacancy color centers show a number of characteristics that make them suitable for implementation in single photon source applications. They operate at room temperature, they do not bleach, they fluoresce under pulsed excitation (they are on-demand), and they are measured to have high quantum efficiencies [3]. All of these characteristics make nitrogen vacancy nanodiamonds promising single photon sources in the field of nanophotonics and quantum communications.

**Semiconductor Quantum Dots**

Another type of single photon source is semiconductor nanocrystals, also known as quantum dots. The energy level structure of quantum dots arises from the quantum confinement effect on bound electron-hole pairs [4]. When the size of the nanocrystal approaches the size of its exciton Bohr radius, the electron and hole can be modeled as a particle confined in a box [4], leading to discrete energy levels. Due to the relatively large exciton Bohr radius of quantum dots (~10 nm), these effects become prominent at crystal sizes much larger than that of atoms or molecules.

Quantum dots also show the property of fluorescing different wavelengths based on their size, even if they are made of the same material [4]. Decreasing the size of the quantum dot causes its energy levels to shift to higher values, much like a particle confined to a box will have its energy levels shifted higher if the box is made smaller [4]. Thus different wavelength light can be achieved by preparing samples of quantum dots with varying sizes. Many quantum dots must be operated at cryogenic temperatures, but there are some types that been shown to fluoresce at room temperatures [3].
Photonic Bandgap Materials

There are many applications for single photon sources, including quantum communications and quantum computing [1]. All of these applications involve encoding information through the quantum states of photons—most of the time, the information is encoded based on the polarization. Thus in implementing a single photon source, it is important to have a source that emits photons of desired frequency or polarization. This can be done through the use of photonic bandgap materials [1, 3].

Photonic bandgap materials are crystal structures that have a periodically varying index of refraction [1], leading to the transmission of certain wavelengths and reflection of others. By doping single photon sources in such crystal solutions, emission of photons with definite properties (polarization or frequency) can be guaranteed. This has been demonstrated by Bissell with both quantum dots and nitrogen vacancy nanodiamonds doped in planar-aligned cholesteric liquid crystals, which form a 1-D bandgap for circularly polarized light [3].

IV. Applications of Single Photon Sources

As mentioned above, there are many applications for single photon sources. One of these applications is quantum communications and cryptography [1]. In quantum communications, information is encoded and transmitted based on the polarization of photons. Vertically and horizontally polarized photons represent “0’s” or “1’s” in binary code [1]. The advantage to information encoded in such a way is that it is completely secure—it is impossible to intercept a message that is encoded on a quantum state because doing so would disturb the quantum state of the photon. Thus it would be evident that the message was tampered with upon receiving it.

Another application of single photon sources is quantum computing. Much like communications, photon polarization can represent either a “1” or “0” in binary code. However, one does not know the polarization of a photon until it is measured, that is, it exists in quantum mechanical superposition of all of its possible states [1]. Because of this, multiple calculations can be carried out simultaneously with a quantum computer—a quantum computing system with n qubits is capable of performing $2^n$ calculations at once [1]. This offers extraordinary computing power compared to modern day computers and would compromise all information currently on the internet or public networks.
Entangled Photon Source

I. Introduction to Quantum Entanglement

Quantum entanglement is a phenomenon where pairs or groups of particles interact in such a way that the measurement of quantum state of one correlates relatively to the properties of the others. When a measurement is made on one member of an entangled pair, the other member at any subsequent time regardless of distance is found to have the appropriate correlated value. Quantum entanglement first came about as a criticism to quantum mechanics by Albert Einstein, Boris Podolsky, and Nathan Rosen in a paper describing the EPR paradox. This paper described how in quantum mechanics a pair of quantum systems could be illustrated by a single wave function. This notion had the writers believe that there either was some interaction between the particles, or the information on the outcomes of the measurements was already present in both particles. These notable scientists preferred the conclusion that “local hidden variables” accounted for quantum entanglement, which meant that quantum mechanics was incomplete since there was no room for such variables. Although this paper focused on the inconsistencies with quantum mechanics, entanglement was later verified experimentally and recognized as valid in the scientific community. Twenty nine years later John Bell postulated that quantum mechanics disagree with “local hidden variables” theories. John Bell proposed a mathematical theorem containing inequalities, which if violated would prove impossible for a local hidden variable theory to exist in quantum mechanics. Experiments such as those performed by Alain Aspect have shown violations of Bell’s inequalities, thus invalidating Einstein Podolsky and Rosen’s preferred conclusion of hidden variables [5]. Entanglement gives grounds for quantum information, quantum cryptography, dense coding, teleportation, and quantum cryptography. The basis for these technologies lies in the idea that measurement of one particle has a nonlocal effect on the reality of another distant particle [6].

II. Theory

Quantum mechanics describes entanglement as two particles that cannot be factored into individual states [7]:

\[ |\Psi_{12} > \neq |\Psi_1 > \otimes |\Psi_2 > \]

Furthermore, the EPR paradox rests on the fact that quantum mechanics can assign a single vector state to two quantum systems. This can be illustrated by the following four functions:

\[
\frac{1}{\sqrt{2}} (|H_1 > |V_2 > \pm |V_1 > |H_2 >) \quad , \quad \frac{1}{\sqrt{2}} (|H_1 > |H_2 > \pm |V_1 > |V_2 >)
\]

Where |H> and |V> represent horizontal and vertical polarization states, and the subscripts denote the different spatial nodes in which the photons are collected [6]. In these cases photon pairs are no longer separable, even if distant from one another.

The importance of John Bell’s discovery was to eradicate the possibility of a so called “hidden variable” by showing that no “hidden variable” theory is capable of reproducing the predictions of quantum physics for the two-photon experiment (figure 4). This mathematical proof was later solidified by countless experiments; subsequently
ruling out any theory involving locality or determinism, and providing a means for proving photons were indeed entangled [5].

![Figure 4: EPR Gedanken experiment (two-photon experiment). The thought experiment involved a source which fired an electron-positron pair to two separate spin measurements [7].](image1)

### III. Types of Entangled Photon Sources

The goal of an entangled photon source is for the emission of two photons exhibiting one of the four entangled Bell’s states. In the earliest tests of Bell’s theorem, atomic cascades were used to produce entangled particles. This involves the release of two photons from decay in a single atom [6]. The resulting photons display nonclassical correlations, and an almost certain avoidance of photon bunching and multiple photon pairs. Atomic cascades decrease in correlation if the photon pair is not released back to back, and involves the trapping of an atom which requires bulky equipment [8].

A more modern form of atomic cascades involves the use of a semiconductor source, or more specifically quantum dots. It was postulated that the same decay in an atom resulting in two entangled photons could occur in a semiconductor quantum dot. This method sparked interest due to semiconductor fabrication techniques involving electrical pumping as opposed to optical pump sources. Additionally, the scale of the entangled photon source could be significantly reduced. This method is hindered by the asymmetric nature of quantum dots, resulting in inconsistencies in the direction of the emitted photons. To account for the asymmetries found in the quantum dots, mechanical strain and an electric field is applied to the semiconductor (figure 5) [8].

![Figure 5: Quantum dot cascade producing two entangled photons through mechanical strain and an electric field [8].](image2)
The most popular form of entangled photon source is through spontaneous parametric downconversion (SPDC). This method involves a pump incident on birefringent crystals which produces the following wave function:

\[ |\Psi_{SPDC}\rangle = |V_1\rangle |V_2\rangle + e^{i\phi} |H_1\rangle |H_2\rangle \]

Compensation using a quartz plate along with a normalization produces an efficient entangled state. The nonlinear crystal produces a signal and idler beam, conserving both momentum and pump energy (figure 6) [7][9].

![Figure 6: Spontaneous parametric downconversion conservation of momentum and energy [7].](image)

By using two nonlinear crystals in perpendicular orientations, two superimposed cones of differing polarization are produced (figure 7)

![Figure 7: nonlinear crystals producing |VV> and |HH> polarization cones [7].](image)

This method requires a strong pump source, due to a photon converting to two longer wavelength entangled photons [9].

**IV. Applications of Entangled Photon Sources**

Quantum entanglement is central to most quantum communications applications. Limited by the efficiency of the entangled photon source, the ability to produce paired...
photons could result in quantum communications through the use of satellites at distances on the 100km scale [10].

In quantum cryptography, entanglement ensures secure key generation between parties sharing entanglement. By using Bell’s inequalities to act as a security test, both parties can ensure that their keys are entangled [9][10].

Another application for entanglement is with superdense coding. This involves sending of information through qubits as opposed to bits. A qubit is a unit of quantum information, such as the polarization of a photon, and as opposed to a bit can be 0, 1, or a superposition of both. If a sender and receiver of qubits are entangled, two classic bits per qubit can be achieved, effectively doubling efficiency [9].

A more farfetched application of quantum entanglement is teleportation. Teleportation of a quantum state can be achieved through entanglement between sender and receiver, where the sender performs a joint Bell state measurement on a desired photon in a certain quantum state (figure 8). This projects the desired quantum state onto the joint pair, and the results of the joint Bell state measurement is sent through classic information to the receiver. The receiver will then be able to reconstruct the initial state of first photon [9].

Figure 8: Quantum teleportation of photon 1 from sender (Alice) to receiver (Bob).
References

Note
James MacNeil – Single Photon Source
Madhu Ashok – Entangled Photon Source