Spectral Properties of Angled-Grating High-Power Semiconductor Lasers

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Abstract—In this paper, we study the spectral properties of angled-grating high-power semiconductor lasers, also known as $\alpha$-distributed feedback (DFB) lasers. We have derived a closed-form expression to describe the cavity resonance. The results of this model are shown to compare favorably with experimental data. Intrinsic device parameters such as coupling coefficient and grating period are shown to be correlated to spectral and near-field characteristics. The formulations and insights developed in this paper allow one to calculate these critical design parameters for optimum performance.

Index Terms—$\alpha$-DFB, angled grating, grating waveguide, high-power semiconductor laser, tilted grating.

I. INTRODUCTION

The compact size, reliability, and efficiency of semiconductor lasers make them ideally suited for a wide variety of applications requiring high power levels. However, conventional wide-stripe semiconductor lasers suffer from poor beam quality due to filamentation and weak lateral mode control. Several techniques have been studied in the past to increase the stripe width without compromising the lateral beam quality [1]–[3]. Recently, gratings have been used in a variety of ways to control the lateral mode [4]–[6]. Of particular interest is the angled-grating laser, also known as the $\alpha$-distributed feedback (DFB) laser. These lasers have been shown to have impressive beam qualities and moderately high power levels through wide output apertures. For example, diffraction-limited output with less than 0.2° lateral divergence has been achieved at output power levels of about 1 W [6]–[8].

Given its impressive performance, it is important to understand its cavity resonance and, in particular, its spectral properties for potential applications in direct modulated communications, as a tunable source and in injection-locking configurations.

The top-view geometry of this device is shown in Fig. 1. The gain stripe is shaped like a parallelogram several hundred micrometers wide by a few millimeters long. In order to suppress Fabry–Perot oscillations, the tilt angle $\theta$ has to be greater than at least $\tan(2\mu/L')$. A grating is etched along the length of the cavity and oriented at an angle to the facets. As depicted in Fig. 1, one can simplistically view the optical waves as continuously bouncing from side to side due to Bragg reflection as they propagate along the length of the cavity. The top and bottom facets reflect part of these waves back into the cavity to form the resonant mode. This resonant mode can be thought of as a snake mode that takes a winding path between the two facets. However, as the theory is developed later in this paper, it will become evident that such a simplistic ray picture is not strictly correct. The field inside the cavity is, in fact, due to the beat pattern between a set of resonant modes. A correct description, therefore, requires a coupled-wave formalism as described in this paper. Furthermore, it should be remarked that, although these resonant modes resemble DFB modes, they are different in several aspects. For example, DFB modes resonate with a detuning on either side of the Bragg wavelength with a characteristic stopband, while these modes resonate in a band of wavelengths centered around the Bragg wavelength, similar to the behavior of a distributed Bragg reflector (DBR) laser.

The angled-grating laser is a hybrid combination of DFB and Fabry–Perot (F–P) lasers. It exhibits characteristics of a DFB cavity due to the evenly distributed grating, but it also exhibits characteristics of an F–P cavity because the facets are essential for resonance. Furthermore, only those waves that are nearly normal to the facet will be efficiently coupled back into

Fig. 1. The angled-grating device configuration to illustrate the simplified view of optical waves bouncing between the grating and the facets. The shaded region indicates the gain stripe, and $\theta$ is the facet tilt angle.
the cavity. If we focus our attention on the waves whose lateral components are at the Bragg wavelength, it can be shown that only a few modes almost perfectly normal to the facet will satisfy the lasing condition. As a result, the phase fronts of the output beam will be parallel to the facet and the beam divergence will be limited only by diffraction from the output aperture. This is a very desirable quality in high-power lasers and difficult to achieve in conventional broad-stripe lasers.

Despite its simple configuration, very little has been published about the operation of this laser. Lang [8] recently studied the lateral mode properties of grating waveguides, finding that they have extremely high spatial mode selectivity. In this paper, we develop a more rigorous theoretical model of an angled-grating oscillator and compare the results with experimental data. Our formulations allow one to calculate critical design parameters such as coupling coefficient, grating period, and cavity lengths for optimum device performance. In this paper, we have focused our attention primarily on the spectral characteristics of the angled-grating laser, although the equations derived here can be equally applied to studying lateral spatial profiles.

The paper is organized as follows. In Section II, the governing equations for cavity resonance are derived using a two-dimensional (2-D) coupled-wave theory. In Section III, experimental and modeling results are presented. In Section IV, these results are compared and contrasted with conventional DFB and F-P lasers. We also compare the analytical formulation with a fully numerical scheme using the beam propagation method (BPM). Finally, concluding remarks are given in Section V.

II. THEORY

A. Formulation of the Coupled Wave Equations

For the convenience of formulating the equations, consider the orientation shown in Fig. 2. Here, the $z$ and $x$ axes are parallel and perpendicular to the grating, respectively. The electrically pumped region is a wide stripe of width $2w$ and length $L$, as shown by the shaded region. Comparing Figs. 1 and 2, $z = z/\cos(\theta)$, and $w = w/\cos(\theta)$.

The coupled wave equations are derived from the 2-D steady-state scalar wave equation

$$\nabla^2 E = -k_0^2 n(x, z) E$$

where $k_0$ is the free-space wave vector and $n(x, z)$ is the refractive index profile using the effective index approximation along the $y$ (epitaxial) direction. The grating and material gain will be treated as a perturbation to the background index $n_0$. Hence, it can be expressed as

$$n(x, z) = n_0 + \Delta n \sin(Kx) + jn_0$$

where $\Delta n$ is the magnitude of the index corrugation and $n_0$ is the imaginary part of the index such that

$$\Delta n \ll n_0$$

$$n_0 \ll n_0.$$

$K$ is the grating number given by

$$K = \frac{2\pi}{\Lambda}$$

where $\Lambda$ is the grating period.

The electric field will be assumed to consist of counterpropagating waves along $+z$ and $-z$

$$E = F(x, z)e^{-j kz} + F'(x, z)e^{j kz}.$$

The spatial profiles of the forward and backward propagating waves will be written as

$$F(x, z) = a_1(x, z)e^{-j kz x} + a_2(x, z)e^{j kz x}$$

$$F'(x, z) = a_1'(x, z)e^{j kz x} + a_2'(x, z)e^{-j kz x}$$

where $k_x$ and $\beta$ are the lateral and longitudinal propagation constants of these trial functions such that

$$k_x^2 + \beta^2 = n_0^2 n_0^2.$$

Substituting (2) and (6) into the wave equation and utilizing the slowly varying approximations

$$\frac{\partial^2 a_1}{\partial z^2} \ll \beta \frac{\partial a_1}{\partial x}$$

$$\frac{\partial a_2}{\partial z} \ll k_x \frac{\partial a_2}{\partial x}$$

results in the following four coupled wave equations:

$$k_x \frac{\partial a_1(x, z)}{\partial x} + \beta \frac{\partial a_1(x, z)}{\partial z} = \frac{k_g}{2} a_1(x, z)$$

$$k_x \frac{\partial a_2(x, z)}{\partial x} - \beta \frac{\partial a_2(x, z)}{\partial z} = \frac{k_g}{2} a_2(x, z)$$

$$k_x \frac{\partial a_1'(x, z)}{\partial x} + \beta \frac{\partial a_1'(x, z)}{\partial z} = \frac{k_g}{2} a_1'(x, z)$$

$$k_x \frac{\partial a_2'(x, z)}{\partial x} - \beta \frac{\partial a_2'(x, z)}{\partial z} = \frac{k_g}{2} a_2'(x, z)$$

Fig. 2. Structure of the device in an orientation convenient for the formulation. $\theta$ is the facet tilt angle.
\[
\begin{align*}
  k_x & \frac{\partial d'_1(x, z)}{\partial x} - \beta \frac{\partial d_1(x, z)}{\partial z} = \frac{k_q}{2} d_2(x, z) \\
  &= k_k e^{2kz} d'_1(x, z)
\end{align*}
\] (15)

where

\[
  k = k_k n_k.
\] (16)

Equations (12) and (13) describe the coupling between the forward propagating modes, while (14) and (15) describe the coupling between the backward propagating modes. We have defined the detuning factor \( \delta \), the coupling coefficient \( \kappa \), and the gain coefficient \( g \) as

\[
\begin{align*}
  \delta &= k_x - K/2 \\
  \kappa &= k_n \Delta n/2 \\
  g &= 2k_n \eta_k
\end{align*}
\] (17) (18) (19)

Due to the 2-D nature of this problem, (12)–(15) are somewhat different from the standard DFB coupled wave equations [9]. However, in the absence of the \( z \) direction, or by setting \( \partial/\partial z \rightarrow 0 \), it can be verified that the above expressions indeed reduce to the standard one-dimensional DFB coupled wave equations.

**B. Solution of the Coupled Wave Equations**

Equations (12)–(15) can be solved by first writing them as second-order differential equations in only one variable. For example, \( a_1(x, z) \) and \( a_2(x, z) \) can be written as

\[
\begin{align*}
  k_x^2 \frac{\partial^2 a_1(x, z)}{\partial x^2} - \beta^2 \frac{\partial^2 a_1(x, z)}{\partial z^2} \\
  - j2\delta k_x \frac{\partial a_1(x, z)}{\partial x} + \left[ \beta g - j2\delta k_x \right] \frac{\partial a_1(x, z)}{\partial z} \\
  + \left[ j\delta k g k_x - \frac{k^2 g^2}{4} - k^2 k_x^2 \right] a_1(x, z) = 0 \\
\end{align*}
\] (20)

with similar expressions for \( a'_1(x, z) \) and \( d'_1(x, z) \). These can be solved by separation of variables and standard solution techniques. For example, (20) can be separated as

\[
\begin{align*}
  k_x^2 \frac{\partial^2 a_1(x, z)}{\partial x^2} - j2\beta k_x^2 \frac{\partial a_1(x, z)}{\partial x} \\
  + \left[ k^2 \kappa^2 + \frac{k^2 g^2}{4} - j\delta k g k_x \right] a_1(x, z) = 0
\end{align*}
\] (22)

along the \( x \) direction and

\[
\begin{align*}
  \beta^2 \frac{\partial^2 a_1(x, z)}{\partial z^2} + \left[ j2\delta k z - \beta g \right] \frac{\partial a_1(x, z)}{\partial z} \\
  + \left( \chi + 1 \right) \left[ k^2 \kappa^2 + \frac{k^2 g^2}{4} - j\delta k g k_x \right] a_1(x, z) = 0
\end{align*}
\] (23)

along the \( z \) direction. The solution of these differential equations is fairly straightforward. They can be compactly expressed as

\[
\begin{align*}
  E &= [A_1(x)e^{i\gamma_1(x)+w} + B_1(x)e^{-i\gamma_1(x)+w}][k_k /2 \beta - j2\delta k x) + k^2 \gamma_1(x)] \\
  + [A_2(x)e^{i\gamma_2(x)+w} + B_2(x)e^{-i\gamma_2(x)+w}]
\end{align*}
\] (24)

where \( A_1(x) \) and \( A_2(x) \) are the lateral fields associated with the forward propagating modes and \( A_1(x) \) and \( B_2(x) \) are the lateral fields associated with the backward propagating modes. These fields can be written as

\[
\begin{align*}
  A_1(x) &= A_1(x)e^{j(K/2)x} + B_1(x)e^{j(K/2)x} \\
  A_2(x) &= A_2(x)e^{-j(K/2)x} + B_2(x)e^{-j(K/2)x}
\end{align*}
\] (25) (26)

Furthermore, reminiscent of conventional DFB formulations, we have defined the following quantities:

\[
\begin{align*}
  \gamma_1 &= \frac{1}{k_x} \sqrt{\delta^2 k_x^2 + \chi_1 \left( \frac{k^2 \gamma_1^2}{4} - j\delta k g k_x \right)} \\
  \alpha_1 &= \frac{1}{\beta} \sqrt{\delta^2 k_x^2 + (1 + \chi_1) k^2 \kappa^2 + \chi_1 \left( \frac{k^2 g^2}{4} - j\delta k g k_x \right)}
\end{align*}
\] (27) (28)

\[
\gamma_1 = \frac{k_k}{\gamma_1 k_x + \alpha_1 \beta}
\] (29)

where \( \chi_1 \) is the complex eigenvalue. A similar set of relations can be obtained for \( B_1(x) \), with corresponding values for \( \gamma_2, \alpha_2, \gamma_2, \) and \( \chi_2 \).

Similar expressions can also be derived for the backward propagating modes \( M_1(x) \) and \( M_2(x) \). However, for the sake of brevity, from now on we will only show the expressions for the forward traveling modes.

Since there is no external excitation or feedback from outside the device, the boundary conditions will be set by requiring that the incoming fields at \( x = \pm w \) are zero, such as

\[
\begin{align*}
  A_1(-w) &= 0 \\
  B_1(-w) &= 0 \\
  A_2(-w) &= 0 \\
  B_2(-w) &= 0
\end{align*}
\] (30) (31)

Applying these boundary conditions to (25) and (26) results in the following two resonance conditions:

\[
\begin{align*}
  \tau_1^2 e^{-j\gamma_1 w} &= 1 \quad \text{for} \quad M_1(x) \quad \text{and} \quad \tau_2^2 e^{j\gamma_2 w} = 1 \quad \text{for} \quad M_2(x)
\end{align*}
\] (32) (33)

The resulting lateral fields can be written as

\[
\begin{align*}
  A_1(x) &= j\gamma_1 A_1 e^{-j\gamma_1(x+w)} - e^{-i\gamma_1 x} \\
  B_1(x) &= A_1 e^{-j\gamma_1 x + \gamma_1(x+2w)} \\
  A_2(x) &= j\gamma_2 B_2 e^{-j\gamma_2 x - e^{-j\gamma_2(x+2w)}} \\
  B_2(x) &= B_2 e^{-j\gamma_2 x - \gamma_2(x+2w)}
\end{align*}
\] (34) (35) (36) (37)

where \( A \) and \( B \) are constants to be determined from the longitudinal boundary conditions.

Solving (32) and (33) results in a discrete set of complex roots \( \chi_1^2 \) and \( \chi_2^2 \), each corresponding to the \( n \)th order of
the lateral mode. In this paper, we will only consider the fundamental mode and, therefore, omit the notation \( n \) from \( \lambda_1^0 \) and \( \lambda_2^0 \). Furthermore, it can be shown that \( \lambda_1 \) and \( \lambda_2 \) are, in fact, complex conjugate pairs. Therefore, the field quantities \( M_1(x) \) and \( M_2(x) \) will also be complex conjugate pairs. We will write these as

\[
\begin{align*}
\lambda_1 &= \chi_r + j\chi_i \\
\gamma_1 &= \gamma_r + j\gamma_i \\
\alpha_1 &= \alpha_r + j\alpha_i \\
\rho_1 &= \rho_r + j\rho_i
\end{align*}
\]

(38)  (39)  (40)  (41)

where all the quantities on the right side of the equations are real.

From (24), one can recognize that \( M_1(x) \) and \( M_2(x) \) have different propagation constants. Therefore, these modes will beat along the \( z \) direction producing a periodic interference pattern, much like the beat pattern of two waveguide modes. If we label the left and right traveling wave components as \( E_l(x, z) \) and \( E_r(x, z) \), respectively, we can show that these components can be expressed as a function of this interference

\[
\begin{align*}
E_l(x, z) &= [B_1(x)e^{j\alpha_1 z} + B_2(x)e^{-j\alpha_2 z}] \\
E_r(x, z) &= [A_1(x)e^{j\alpha_1 z} + A_2(x)e^{-j\alpha_2 z}]
\end{align*}
\]

(42)  (43)

As a result, both \( E_l(x, z) \) and \( E_r(x, z) \) will have a slowly varying envelope along the \( z \) direction with a period of \( 2\pi/Re\{\alpha_1 + \alpha_2\} \). It is this oscillation that results in the snake-like appearance of the optical beam. Similarly, \( \alpha_2 \) will be the extinction coefficient along the \( z \) direction. This extinction is due to the power that leaves the cavity laterally through the grating.

We will define a characteristic exchange length \( L_e \) as half the beat length between \( M_1(x) \) and \( M_2(x) \). This is the length in which the power in \( E_l(x, z) \) is transferred to \( E_r(x, z) \). This length can be written as

\[
L_e = \frac{\pi}{Re\{\alpha_1 + \alpha_2\}} = \frac{\pi}{2\alpha_r}.
\]

(44)

In Fig. 3(a) and (b), contour plots of the magnitudes of \( E_l(x, z) \) and \( E_r(x, z) \) are shown. These were calculated by assuming a transparent medium \( (g = 0) \). The data in Fig. 3(a) were obtained by plotting (42) and the data in Fig. 3(b) were obtained by plotting (43). Other parameters used for this calculation are shown in the figure caption. As expected, one can clearly see the periodic beat pattern in these figures. Note that in Fig. 3(a) the intensity is zero at \( x = +w \) and nonzero at \( x = -w \). This represents power flow along the \( -z \) direction. Similarly, in Fig. 3(b) the intensity is zero at \( x = -w \) and nonzero at \( x = +w \). This represents power flow along the \( +z \) direction. In Fig. 3(c), the magnitude of the total field is shown. This is the snake mode of the structure. It was obtained by plotting (24).

### III. FACET REFLECTION

Fig. 4 depicts the reflection from the bottom facet. Using the notation from Fig. 2, the incident and reflected waves will be represented as \( E'_i(x, z\tan\theta) \) and \( E'_{r}(x, z\tan\theta) \), respectively.

\[
2\beta \hat{k} \sin(\theta - \theta_B) x
\]

(45)

We will assume that the incident wave is a resonant mode of the structure with a lateral propagation constant of \( K'/2 \) and that it is slightly off-normal to the facet. The angle of this wave with the grating will be represented as \( \theta_B \), as shown in Fig. 4. It can be seen that only for the special case of \( \theta_B = \theta \) will the phase fronts of the incident and reflected waves be parallel to the facet. In general, depending on the wavelength, the phase fronts will be slightly misaligned with the facet. This will lead to reflection losses as power will be coupled into higher order and radiation modes. This misalignment can be accounted for in the reflection coefficient by multiplying the incident field by a phase factor

\[
\text{and performing an overlap integral with the reflected mode } E'_i(x, z\tan\theta) \text{. The treatment of this problem is similar to that}
\]
of a waveguide mode reflecting off a tilted facet [10]. The resulting reflection coefficient can be written as

$$\mathcal{R}_f = \frac{\eta_0}{2c_0\mu_0 P_o} \int_{-\infty}^{\infty} \left| E'_f(x, x \tan \theta) E_f(x, x \tan \theta) \right| \cdot e^{2\pi i \sin(\theta \cdot \delta_p) x} \, dx$$

(46)

where $P_o$ is the power in the incident mode and $\mu_0$ and $c_0$ are the magnetic permeability and speed of light in vacuum, respectively. As a result, the power reflection coefficient becomes $r_f^2 \mathcal{R}_f^2$, where $r_f$ is the Fresnel reflection coefficient of a plane wave that is reflected from a tilted facet. The tilt angles we will be considering here are all nearly very normal to the facet. Furthermore, $r_f$ is a much weaker function of tilt angle compared to $\mathcal{R}_f$. Therefore, we will neglect the dependence on $r_f$ on tilt angle and only consider the dependence of $\mathcal{R}_f$.

From (46), it can be seen that, for perfectly normal incidence, the magnitude of $\mathcal{R}_f$ will be dictated only by the mismatch between the field distributions of $E'_f(x, x \tan \theta)$ and $E_f(x, x \tan \theta)$. In most cases, this mismatch is extremely small such that the overlap between the two fields can be taken to be unity. As the incident mode becomes more and more off-normal, $\mathcal{R}_f$ will get progressively smaller. When $\theta$ deviates from $\theta_B$ by a large amount, $\mathcal{R}_f$ will quickly vanish and become zero. As a result, only a small cone of angles around normal incidence will be successfully reflected by the facet into the resonant mode. It will be shown later that this cone angle is so small that the divergence of the output beam is limited by the diffraction from the output aperture of the laser.

Similarly, at the top facet, $E'_f(x, L + x \tan \theta)$ will reflect and couple into $E'_f(x, L + x \tan \theta)$ and $E_f(x, x \tan \theta)$ to be nearly normal to the facets, we have implicitly assumed $E_p(x, L + x \tan \theta)$ and $E'_f(x, x \tan \theta)$ to have large incidence angles, of the order of $2\theta$. Following the same arguments as before, it can be shown that these modes will be very poorly coupled into the reflected modes. Therefore, while the first two modes remain nearly normal to both facets and contribute to resonance, their complementary pairs will impinge the facets at a large angle and will quickly exit the cavity. Hence, we can approximate

$$E'_f(x, L + x \tan \theta) = 0 \quad (47)$$

$$E_f(x, x \tan \theta) = 0. \quad (48)$$

IV. Cavity Resonance

Combining all of the above equations results in the following complete cavity resonance equation:

$$\left[ \frac{1}{2} \left( 1 + \cos(2\alpha_0 L) \right) \right] \cdot \left[ e^{-2\alpha_0 L} \cdot \mathcal{R}_f \right]^2 \mathcal{R}_f^2 = 1.$$  

(49)

This equation embodies the entire spectral and threshold behavior of the device. The terms inside the first square brackets represent the power oscillations along the longitudinal direction as a result of the beating modes $M_1(x)$ and $M_2(x)$. If the cavity length is an exact even multiple of the exchange length $L_c$, then the cosine term would become unity, reducing the first square brackets to unity. If the cavity length is not an exact even multiple of $L_c$, then $E'_f(x, L + x \tan \theta)$ and $E'_f(x, x \tan \theta)$ will be nonzero at the facets. Since these waves do not get reflected back into the cavity, they will eventually exit the cavity and will contribute to the overall cavity loss. This cavity loss appears as a reduced magnitude of the terms inside the first square brackets. If the cavity length is an odd multiple of $L_c$, then this term reduces to zero, implying that such a laser would have a prohibitively high threshold gain. The terms inside the second square brackets represent the longitudinal loss coefficient. This loss is due to the lateral power loss through the grating. The terms inside the third square brackets represent the reflection at the facets. $r_f$ represents the Fresnel reflection coefficient of a plane wave at the facets, and $\mathcal{R}_f$ represents the effect of the tilted phase fronts. Finally, the terms inside the last square brackets represent the material gain and the Fabry–Perot-like longitudinal resonance. These terms produce a finely spaced multimode spectrum within the envelope defined by all the previous terms. The value of $g$ that satisfies (49) both in magnitude and phase will be the threshold gains for the various longitudinal modes.

V. Experimental and Theoretical Results

The spectra of several 980-nm lasers were examined at below- and above-threshold current levels. Typical spectra measured at 700 mA and 1.0 A are shown in Fig. 5. In all of the below-threshold measurements, we consistently observed the Fabry–Perot like oscillations confined under a narrow spectral envelope about 4 nm wide. When pumped slightly above threshold, the spectrum collapsed to a single-mode output, as shown in Fig. 5(b). We attribute this to the small differences in threshold gains of these modes and to homogeneous broadening of the gain spectrum.

Despite this decidedly single-mode operation, care must be taken in defining the angled-grating laser as a single-mode laser [8]. Although exhibiting much better side-mode suppression than conventional Fabry–Perot lasers under CW injection, they are not dynamically single mode. Under pulsed conditions, they operate transiently in a multimode spectrum. For example, Fig. 6 shows the spectral output of the same laser at 1.0 A with 500-ns pulses at 1% duty cycle. It is clear that the side-mode suppression is much lower than in Fig. 5(b), with many modes oscillating throughout the course of the current pulse. This behavior is typical of lasers having relatively small threshold gain differences between adjacent modes. This inference is further supported by the observation of mode hopping as the laser was temperature tuned. Fig. 7 shows a plot of lasing wavelength versus submount temperature for an angled-grating laser operating CW at 1.6 times its threshold current. Solid circles indicate the wavelength of an angled-grating laser operating CW at 1.6 times its threshold current. Solid circles indicate the wavelength of the laser emission as the temperature was increased, while open circles locate the wavelength when the temperature was subsequently decreased. Arrows denote mode hops, where the mode hop separations are consistent with Fabry–Perot mode spacings associated with the laser length $L'$. Hysteresis is evident in the tuning path, which has a slope of approximately 0.78 Å/°C. Fig. 7 was fairly representative behavior for the
Fig. 5. Measured spectra (a) below threshold (700 mA) and (b) above threshold (1.0 A) under CW conditions. The mode spacing in the below-threshold spectrum is about 0.09 nm.

several devices examined, although some tuned nearly 1.5 nm without mode hopping at specific operating currents. This large tuning range can be understood by recognizing that the Bragg wavelength of the grating and the Fabry–Perot resonances tune at the same rate, thereby maintaining a fixed side-mode suppression with temperature connected with the cold-cavity resonator. In general, however, the observed mode hopping and multimode transient spectra indicate that, unlike conventional DFB lasers, the angled-grating laser is not a dynamically single-mode laser.

Fig. 8 shows the calculated threshold gains of the longitudinal modes for different cavity lengths. These were obtained by solving for $g$ in (49). For this calculation, we have used $20^\circ$ for the grating tilt and $2w' = 140 \mu m$ for the width (or $2w = 149 \mu m$ if measured along the $x$ axis). The facets were assumed to be HR/AR coated at 95% and 5%. In order to fit the experimental data for the 1250-μm-long device, we have assumed a $\Delta n$ of $5.1 \times 10^{-3}$, a grating period of 449.169 nm, a mode index of 3.20, a dispersion factor $dn/d\lambda = 1.15 \mu m^{-1}$, and a uniform material scattering loss of $3.0/cm$. It should be noted that the calculation disregards spatial-hole burning, carrier-induced index change, and other such effects. These approximations are quite reasonable for the purposes of calculating threshold gains and below-threshold spectra. As expected, one can clearly see several finely spaced Fabry–Perot-like modes confined under a narrow spectral envelope. The mode spacing for the 1250-μm-long device is approximately 0.09 nm, which is consistent with experimental measurements. The threshold gain difference between adjacent modes is about 0.1/cm at the lowest point (which also corresponds to the facet normal case). Combined with the gain roll-off effect, this results in better mode discrimination than conventional Fabry–Perot lasers [9]. Furthermore, the threshold gain is strongly dependent on cavity length. This arises due to the mismatch between the cavity length and the exchange length $L_c$. For optimum results, one has to cleave the cavity length to be as close as possible to an even integer number of exchange lengths.

Fig. 9 shows the below-threshold spectrum calculated from the left side of (49) following the approach described in [11] for $g = 160/cm$. The envelope of this spectrum is qualitatively consistent with the measured below-threshold spectrum shown in Fig. 5.

Since (49) is the central result of this paper, it is instructive to examine each group of terms within the square brackets separately. These are shown in Fig. 10. All the device pa-
Fig. 8. Calculated threshold gain from (49). The points indicate the resonant wavelengths, but they have been connected by the lines to show the trend. Cavity lengths shown are the perpendicular distances between the facets. For the lengths along the $z$ axis, they have to be divided by $\cos(20)$.  

Fig. 9. Calculated below-threshold spectrum from (49) using a gain of $g = 16.0/\text{cm}$. The wavelength spacing is approximately 0.09 nm.

Fig. 10. The wavelength dependence of (a) the facet power coupling term $R_f^2$, (b) incidence angle with the facet, (c) mismatch between the exchange length and the cavity length, and (d) the longitudinal power loss. These correspond to the terms in (49). The circles are numerically calculated results for $R_f^2$ using the finite-difference beam propagation method.

rameters were the same as given earlier, and a cavity length of $L' = 1250 \ \mu\text{m}$ was used. In Fig. 10(a), the wavelength dependence of the facet power coupling term $R_f^2$ is shown. In this example, the resonant mode is exactly normal to the facet at $\lambda_0 = 983.2 \ \text{nm}$. As the wavelength deviates from this value, the resonant mode becomes more and more off-normal with the facet, and the fraction of the power that is coupled into the resonant mode gets smaller. In this case, the FWHM bandwidth of $R_f^2$ is approximately 3.8 nm. Furthermore, it is interesting to note that this bandwidth would get smaller as the stripe gets wider, implying improved spectral selectivity. Despite the off-normal incidence, the output beam remains well within the diffraction limit of the output aperture. This can be demonstrated by calculating the angle of incidence of the wave fronts as a function of wavelength. This is shown in Fig. 10(b). It can be seen that the phase fronts are less than 2 mrad off-normal throughout the wavelength range of interest. This is about four times smaller than the diffraction limit of the output aperture. In Fig. 10(c), the terms within the first square brackets of (49) are shown. These terms represent the mismatch between the cavity length and the exchange length $L_e$. The exchange length is a weak function of wavelength, varying slowly from 82.4 to 83.6 $\mu\text{m}$ between 975 and 992 nm. At the lasing wavelength of $\lambda_0 = 983.2 \ \text{nm}$, the cavity length ($L' = 1250 \ \mu\text{m}$) is $16.0L_e$. This results in a nearly perfect cavity-matching condition. In Fig. 10(d), the extinction term $e^{-2\pi L_e}$ is shown. This represents the inherent longitudinal power loss of the modes due to lateral transmission loss through the grating. This term is only weakly dependent on the wavelength and can be taken to be a constant throughout the range of wavelengths considered. Within the envelope defined by all of the above terms, the longitudinal phase oscillations will traverse through many $2\pi$ [phase term of the fourth brackets of (49)], producing the finely spaced multimode spectrum as seen on Fig. 5. Hence, the spectral output from this device consists of multiple longitudinal lines confined under a narrow passband.

In order to numerically validate the expression for $R_f^2$, we compared these results with a fully numerical finite-difference beam propagation method. The fractional amount of power remaining in the resonant mode was calculated for several wavelengths after a propagation distance of 2 mm. $R_f^2$ was calculated after accounting for the longitudinal power loss. The results are shown as the closed circles in Fig. 10. It can be seen that these points agree quite well with the analytical results. The implementation of the beam propagation method is discussed in more detail in the following section.

VI. DISCUSSION

A. Comparison with BPM

It is particularly insightful to compare the analytical formulations derived in this paper with a more rigorous numerical scheme. For this purpose, we have implemented a one-way beam tracing algorithm using the paraxial finite-difference beam propagation method (FD-BPM) [12]. A computational grid spacing of $\Delta x = 0.02 \ \mu\text{m}$ and $\Delta z = 0.02 \ \mu\text{m}$ was used, and all the other device parameters were the same as
Fig. 11. Beam propagation plots for (a) 986 nm and (b) 983.2 nm (facet normal case) assuming a transparent medium.

given earlier. The resonant field profile $E_r(x)$ was computed using (43) at 986.0 and 983.2 nm. These fields were made to reflect at the bottom mirror and then used as the initial field profiles for the beam propagation simulation. The beams were traced through the length of the device assuming a transparent medium. The resulting field intensity profiles are shown in Fig. 11.

Fig. 11(a) corresponds to 986.0 nm. The resonant mode in this case is slightly off-normal to the mirror (about $3 \times 10^{-3}$ rad). Upon reflection, the beam will acquire many higher order and radiation components. This is the initial condition for the simulation. As they propagate, the power in these modes will dissipate rapidly, leaving only the power in the resonant mode. This behavior can be clearly seen in the figure.

For comparison, Fig. 11(b) corresponds to a wavelength of 983.2 nm. Since the resonant mode is perfectly normal to the facets at 983.2 nm, the initial field profile at the bottom of the trace is very close to the exact resonant mode of the structure. As a result, the coupling between the initial field and the resonant mode is nearly perfect, and very little power is lost due to radiation and higher order modes. The power is well contained within the grating as it oscillates from side to side and propagates along the length of the device. One can also notice the transmissions through the grating which appear periodically along both edges of the device, which are indicated by arrows in the figure. Those transmissions that are closest to the facet will make it through to the output and can be detected as weak satellite peaks on the near-field profile. For comparison, a plot of a measured near-field profile is shown in Fig. 12. The satellite peaks in this figure can be used to independently confirm the exchange length. By measuring the distance between these satellite peaks and projecting them along the $z$ axis, the exchange length can be estimated to be approximately 83 $\mu$m. Using the parameters given earlier, (44) also predicts an exchange length of 83 $\mu$m, which further validates our theoretical formulations.

For a quantitative comparison of the beam propagation method and the analytical theory, Fig. 13 shows the fractional lateral power flow as a function of longitudinal distance for three different wavelengths. The solid lines correspond to the beam propagation results, and the dashed lines correspond to...
the theory developed in this paper. In the BPM scheme, this is calculated as

$$P_{x}^{BPM}(z) = \frac{\int_{-\omega}^{+\omega} (\partial E/\partial x)E^*|_{z=0} \, dx}{\int_{-\omega}^{+\omega} (\partial E/\partial x)E^* \, dx}.$$  \hspace{1cm} (50)$$

In the analytical case, this is simply

$$P_{x}^{CMT}(z) = \frac{\int_{-\omega}^{+\omega} \left[|E(x,z)|^2 - |E_r(x,z)|^2\right] \, dx}{\int_{-\omega}^{+\omega} \left[|E(x,0)|^2 - |E_r(x,0)|^2\right] \, dx}.$$  \hspace{1cm} (51)$$

The curves denote the magnitude and direction of the power flow along the lateral direction within the confines of the grating stripe. Positive numbers correspond to a net power flow along the $+\hat{x}$ direction and negative numbers correspond to a net power flow along the $-\hat{x}$ direction. The zeros correspond to the positions where the net power flow along the lateral direction is zero. In the BPM approach, one cannot distinguish between the reflected component and the transmissive component until the transmitted power completely exits the cavity laterally through the grating. Instead, the transmissive component will remain inside the grating for a short distance before exiting. On the other hand, the analytical formulation instantly excludes the transmitted power. This will naturally lead to some difference between the two schemes and is the primary reason for the small difference in amplitude between the two curves in Fig. 13. Furthermore, it is well established that paraxial finite-difference BPM suffers from poor retention of beam directions, especially for off-axial propagations [13]. Even a slight deviation of the beam’s direction can severely disrupt its detuning with the grating. Despite these significant shortcomings, we find the agreement between the two schemes in Fig. 13 to be a good confirmation of the theoretical formulations derived in this paper. In particular, the oscillatory nature of the waves and the power loss due to lateral transmission through the grating agree quite well between the two schemes.

B. Leaky Modes

Fig. 13 also highlights an important limitation of the grating resonant mode picture. As described previously, when the resonant mode is not perfectly normal to the facet, the reflected wave will contain radiation components. The power in these radiation modes is carried through leaky modes. By definition, these modes have larger attenuation coefficients than resonant modes. As seen from Fig. 13, the leaky modes do not completely die off within a short distance. One can notice that there is some power remaining in the radiation modes even as far away as 1 mm from the facet. Strictly speaking, therefore, the resonant mode picture is valid only for long cavities. If the cavity is relatively short, of the order of 1 mm or less, it is possible for longitudinal resonances to be formed via leaky modes. These modes remain normal to the facet at all wavelengths and propagate at slightly different directions than the resonant modes. Being normal to the facet, they suffer less power loss at the facets than the resonant modes. However, being leaky modes, they will incur larger propagation losses through the grating. Hence, there exists a competition between resonant modes and leaky modes. In general, one can expect leaky modes to become favored when their propagation losses become smaller than the facet losses of the resonant modes. This condition will occur when the resonant mode substantially deviates from the facet normal condition, which happens at both extremities of the spectral passband. Therefore, resonant modes will dominate near the center of the passband, while leaky modes will dominate near the edges of the passband.

C. Comparison with a DFB Cavity

It is interesting to note that, unlike conventional DFB lasers, neither the theoretical formulations nor the spectral data exhibit a stopband. Instead, we see a group of longitudinal
modes within a passband around the Bragg wavelength, which is more like the spectral characteristics of a long distributed Bragg reflector (DBR) laser. This can be understood by carefully reviewing the origin of the stopband in a DFB laser.

In a DFB laser, the waves travel normal to the grating. The wavevector of the field envelope inside the cavity is \( \gamma_l = \sqrt{k^2 - \beta^2} \). Therefore, for \( |\beta| < \kappa \) (near the Bragg condition), the waves will be evanescent. Since evanescent waves do not produce resonant states, there will be no modes within a spectral range of \( |\beta| < \kappa \). For \( |\beta| > \kappa \), the waves become complex, and resonant states can be found. Even then, an additional phase-matching condition between the counterpropagating waves must be met. Therefore, although the reflectivity is a maximum at the Bragg wavelength, the first resonant mode occurs for values of \( |\beta| \) slightly greater than \( \kappa \). This is the origin of the DFB stopband.

In the angled-grating laser, the 2-D nature of the structure lends itself to an extra degree of freedom. For any given wavelength, it is possible to form traveling waves at the Bragg wavelength by simply tilting the wave by different amounts. By distributing part of the \( k \) vector along the longitudinal direction and part of it along the lateral direction, resonant modes can always be found for any wavelength. An example of this is shown in Fig. 14, where the exchange lengths are calculated as a function of index corrugation \( \Delta \eta \) for different wavelengths. Despite the widely varying wavelength, it can be seen that resonant modes exist for all those wavelengths. They differ from each other only in their propagation angles \( \theta_B \) and exchange length \( L_e \). In this case, \( \theta_B \) corresponds to 20°, 27°, and 32° for 983, 1300, and 1550 nm, respectively for a grating period of 449.169 nm and a mode index of 3.2. \( \theta_B = \pi/2 \) corresponds to the cutoff wavelength of the grating. In this example, that would correspond to a wavelength of 2.87 \( \mu m \). This extra degree of freedom does not exist in a conventional DFB laser. Since wave propagation is confined along one dimension only, resonances will occur at discrete wavelengths. In a sense, therefore, a lateral grating stripe is similar to a waveguide while a DFB cavity is similar to a resonator.

Furthermore, the longitudinal field distribution in a conventional DFB laser and the lateral field distribution in an angled-grating laser have a finite spatial profile. Naturally, in both cases there will exist a stop-band in \( k \)-space. In a conventional DFB laser, this spatial stopband is directly translated into a spectral stopband. In an angled-grating laser, the extra degree of freedom along the longitudinal direction absorbs the spatial stopband. This allows many wavelengths to propagate through the grating. The spectral filtering in the angled-grating laser is primarily dictated by the angular reflection from the facet. The envelope of this spectral filter is centered around the wavelength that is perfectly normal to the facet. Therefore, we see a passband spectrum where one might expect a stopband.

VII. CONCLUSIONS

In this paper, we have derived a comprehensive theory for angled-grating lasers. Simple expressions for the cavity resonance and the exchange length have been derived. Results from this model have been compared favorably with experimental data, and spectral and near-field characteristics have been related to intrinsic device parameters. The formulations allow us to calculate critical design parameters such as coupling coefficient, grating period, and cavity lengths for optimum performance. Furthermore, this theory has been compared with a fully numerical finite-difference beam propagation approach for confirmation of the derived formulations.

The spectral envelope is dictated primarily by the angular relationship between the resonant mode of the grating and the facets. When the resonant mode is perfectly normal to the facet, the facet coupling factor between the forward and backward traveling waves becomes nearly unity. Therefore, the spectrum is characterized by a passband centered around the facet-normal wavelength. Within this band, there are finely spaced Fabry–Perot-like modes. These modes have a better discrimination than conventional Fabry–Perot lasers and fall within a much narrower envelope. Hence, it is possible to operate the laser as a single mode laser under CW conditions. However, unlike conventional DFB lasers, the angled-grating laser is not dynamically single mode. When injected with a pulsed current, it reverts to multimode operation. Furthermore, the spectrum does not show a stopband, but rather a passband where one might expect a stopband.

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