Phase effects of secondary reflections on the performance of reflective liquid-crystal cells

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Abstract: In this paper, the phase effects of secondary reflections in reflective liquid-crystal cells are explored. The existence of such secondary reflections are analytically predicted and experimentally verified. The wavelength dependence of the phenomena is used to explore the magnitude of the effect. Changes of the net retardation due to these secondary reflections are measured to be +/- 3%. Numerical modeling verifies that the root cause of these changes is secondary reflections, and that as much as 10% change can be obtained for different liquid-crystal cell thicknesses and material compositions. The deleterious effects of these secondary reflections are explored from a device performance perspective, and found to be most harmful for incident light with a broad spectral bandwidth.

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References and Links

8. See, for example, Y.A. Nastishin, R.D. Polak, S.V. Shiyanoervsi, V.H. Bodnar, and O.D. Lavrentovich, “Nematic polar anchoring strength measured by electric field techniques,” J. Appl. Phys. 86, 4199-4213 (1999), and references within.
9. See, for example, Chapter 4 of Reference 7, or Chapter 8 of Reference 10.
1. Introduction

Liquid crystals are effective in changing the polarization-dependent phase of light propagating through the medium as a function of applied voltage. An important ramification of this behavior is the ability to manipulate the polarization state of the light that passes through the medium. When used in conjunction with a polarizer, the output intensity of the light can be electronically controlled. By using tiny arrays of electrodes, the light intensity can be varied across a given aperture, yielding a spatial light modulator [1]. Such spatial light modulators are used in a wide variety of applications, ranging from beam shaping [2, 3] to arrayed optical switching devices [4, 5].

The cell that holds the liquid crystal is comprised of several material layers with varying functionalities, such as anchoring the liquid crystal molecules at the boundaries, making electrical contacts, and having a bulk substrate to work with for deposition and handling purposes. These various layers, which the light must propagate through, have various indices of refraction. The result is an array of interfaces each of which causes minute reflections that add coherently with the original beam. Such reflections together with diffractive effects have been used to explain throughput variations in transmissive liquid-crystal cells having material interfaces that yield large reflections (2-4%) [6].

In this paper, the ramifications of secondary reflections on the phase of the output light in reflection liquid-crystal cells are explored. Throughout this paper, the common parallel-aligned nematic liquid-crystal configuration [7] is used in order to evaluate this effect in a tractable manner, as is often done when investigating new effects in liquid-crystal cells [8]. Secondary reflections will also occur in more complicated liquid-crystal configurations, however more sophisticated optical modeling is required.

2. Secondary reflections

In order to gain some physical insight in understanding the nature of this phenomenon, consider a simplified liquid-crystal cell as a material surrounded by reflective surfaces, as depicted in Fig. 1. The dark blue lines represent the interfaces under consideration, and the light blue region represents the material between the interfaces. For our purposes, this “material” may consist of many different layers, provided we are only interested in examining the isolated effects of the bounding (dark blue) interfaces. This is only reasonable to do if the reflections at the intermediary interfaces are relatively weak, which will be the case for the optical stacks under consideration. For generality, we will consider that either the material or a given layer within the material may be birefringent.

In Fig. 1, the incident interface has complex field reflectivity and transmissivity coefficients $r_1$ and $t_1$, and the reflecting interface has complex reflectivity $r_2$. For the reflective geometry under consideration, $r_1$ is small, and $t_1$ and $r_2$ are near unity. Since the index differences are small, it shall be assumed for instructive purposes that the reflection and transmission coefficients are identical for the ordinary and extraordinary waves. The medium has a single-pass ordinary and extraordinary phase accrual of $\frac{1}{2}\phi_o$ and $\frac{1}{2}\phi_e$ respectively. The incident beam, notated by $\hat{E}_0$, nominally transmits through the first interface, through the medium, reflects off the surface with reflectivity $r_2$, then returns through the medium and exits back through the first interface. This exit field is denoted by $\hat{E}_1$. Consider two additional fields, as depicted in Fig. 1. As the light from the primary pass propagates through the first interface and is exiting the medium, a small reflection gets sent back and experiences a second pass through the medium, accruing additional phase. This field is denoted by $\hat{E}_2$. Similarly, as the field $\hat{E}_0$ initially impinges on the first interface, a small portion is reflected back, denoted as $\hat{E}_{-2}$. Provided the reflections $\hat{E}_2$ and $\hat{E}_{-2}$ are weak, no further reflections need to be considered in this analysis.
First consider the coherent addition of the ordinary components of only $\vec{E}_1$ and $\vec{E}_2$. Assuming only weak reflections from the top surface ($r_1 << 1$), the total reflected field is given by

$$E_\text{r}^\circ = E_1^\circ + E_2^\circ = E_1^\circ r_1 r_2 \exp(i\phi_\text{r})[1 + r_1 r_2 \exp(i\phi_\text{r})],$$

(1)

where $E_1^\circ$ is the ordinary component of the field $\vec{E}_1$. The terms outside the square brackets represent the primary reflected field $E_1^\circ$. The additional term within the brackets represents the effect of coherent addition of the secondary reflection. The bracketed terms can be turned into a single term containing amplitude and phase as

$$E_\text{r}^\circ = E_1^\circ \left[1 + 2\sqrt{R_1 R_2} \cos(\phi_\text{r} + \phi_i) + R_1 R_2 \right]^{1/2} \exp(i\Delta\phi_\text{r}).$$

(2)

where $\phi_i$ is the total phase gained by the reflective interfaces, $R_i = |r_i|^2$, and $\Delta\phi_\text{r}$ is the additional phase accrued by the field due to the secondary reflection. The bracketed amplitude change deviates from unity by an amount proportional to the reflectivity of the first surface. Since $R_1$ is small, this contribution to the exit field will have a negligible effect for the purpose of this analysis.

The additional phase accrual to the net field due to the secondary reflection is given by

$$\Delta\phi_\text{r} = \arctan \left[ \frac{\sqrt{R_1 R_2} \sin(\phi_\text{r} + \phi_i)}{1 + \sqrt{R_1 R_2} \cos(\phi_\text{r} + \phi_i)} \right].$$

(3)

Since the secondary reflection is small, the denominator can be approximated by unity, and the arctangent can be eliminated to yield

$$\Delta\phi_\text{r} = \sqrt{R_1 R_2} \sin(\phi_\text{r} + \phi_i).$$

(4)

A similar equation can be derived for the extraordinary wave. Combining Eqs. (2) and (4) for both ordinary and extraordinary waves, the nominal retardation is modified by an
amount $\Delta \phi_0 - \Delta \phi_e$. Define $\phi'_o = \phi_o + \phi$, and $\phi'_e = \phi_e + \Delta \phi$, where $\Delta \phi$ is the nominal phase difference between the ordinary and extraordinary fields experiencing a single reflective pass through the cell. The residual retardation, caused by the secondary reflection, is given by

$$\rho_2 = \sqrt{\frac{R_1 R_2}{2 \pi}} \left[ \sin(\phi'_o) - \sin(\phi'_e + \Delta \phi) \right],$$

(5)

where $\rho_2$ is in units of waves.

It is clear from Eq. (5) that the strength of the residual retardation is governed by the magnitude of the secondary reflection. If the trigonometric quantities are simply allowed to take on their extreme values and the back surface reflection is approximated as unity, the maximum size of the effect is dictated by $\rho_2 = \sqrt{R_1 / \pi}$. For a power reflection of 0.1%, the residual retardation will fall in the range of +/-10 milliwaves. This large change to the expected retardation may be detrimental to the performance of the device, depending on the application. If a tertiary reflection were considered, the additional change to the retardation would be approximately two orders of magnitude smaller than the contribution due to the secondary reflection due to the two additional small reflections incurred, and is thus negligible.

Equation (5) also shows that the secondary reflection is only important if it makes an additional round trip through the birefringent medium. While it is intuitive that a small or negligible birefringence should likewise produce an insignificant residual retardation, it also implies that secondary reflections which occur between the reflective layer ($r_2$) and the liquid-crystal boundary nearest that layer will have no effect on the net retardation.

The last distinguishing feature of Eq. (5) is the isotropic phase accrual, $\phi'_o$, which affects the residual retardation in several ways. First, the exact material composition and thickness of each layer impacts the value of the residual retardation. Subsequently, variations in the fabrication of the cell will lead to different magnitudes of this effect. The second ramification, which is also manifest in $\Delta \phi$, is the wavelength dependence of the residual retardation. Since the phase accrual through the optical path is wavelength dependent, one would expect the magnitude of this effect to vary as a function of wavelength. This feature will be explored in more detail in Section 3.

In deriving Eq. (5), it was assumed that the secondary reflection came from the exit side of the first surface (i.e., $\vec{E}_{r_2}$). If the reflection from the initial impingement on the surface (i.e., $\vec{E}_{r_2}$) is instead used as the secondary reflection term, the complimentary equation to Eq. (5) is

$$\rho_2 = \frac{1}{2 \pi T_1} \sqrt{\frac{R_1}{R_2}} \left[ \sin(\phi'_o) - \sin(\phi'_e + \Delta \phi) \right],$$

(6)

where $\phi'_o$ embodies the required reflective and transmissive phases due to the interfaces. Since $R_2$ and $T_1$ are approximately unity, Eq. (6) yields the same magnitude for the residual retardation as Eq. (5). Like the previous case, this reflection is only significant if the reflected path experiences a different total birefringence than the primary reflection experiences.

The combined effects of secondary reflections on the liquid crystal cell will of course be a coherent combination of all reflections at every interface within the cell. This type of multi-layer stack modeling will be described and used in the analysis in Section 3.
3. Experimental data and theoretical comparisons

The liquid-crystal cell used for our measurements is schematically shown in Fig. 2(a). The cell contained parallel-aligned nematic liquid-crystal molecules anchored by polyimide layers, yielding a net birefringence of $n_e - n_o = 0.11$. ITO formed the transparent electrical contact of the reflective cell, and gold formed the reflective contact. The entire structure was bound by fused silica substrates. The refractive index steps between the layers ranged from 0.1-0.2, yielding power reflection levels from 0.1-0.5%.

Data was gathered from the liquid-crystal cell by interrogating the polarization state in the amplitude modulation or spatial light modulator (SLM) geometry, as shown in Fig. 2(b). The liquid-crystal cell is preceded by a linear polarizer and a nominally quarter-wave plate, with the required difference from quarter wave being the residual single-pass retardation of the liquid-crystal cell. The light is normally incident polarized along the axis of the polarizer. The slow axis of the waveplate and the extraordinary axis of the liquid crystal cell are parallel and aligned at a 45-degree angle with respect to the linear polarizer. The output of the system shown in Fig. 2(b) ranges from transmission mode to extinction mode, depending on the voltage level applied to the cell.
The initial indicator of a wavelength-dependent phenomenon is born out by investigating the liquid-crystal cell in the SLM geometry. Scanning the output of light incident on this configuration as a function of voltage yields the condition for maximum extinction. Since the birefringence experienced by the optical signal is dependent on its wavelength, one would expect an approximately linear dependence on the extinction voltage as a function of wavelength. However, Fig. 3 demonstrates that this is not the case. There is in fact a rather strong non-linear dependence of the extinction voltage on the input wavelength. This data is an indication of the effects of secondary reflections.

Continuing to examine the wavelength dependence of this phenomenon, let us consider the net retardation of the solitary liquid-crystal cell as a function of voltage, since this is the primary quantity of interest that is being modified by the secondary reflection effects, and also the quantity that will most affect the performance of the cell. By measuring the extinction of the SLM geometry at a given wavelength as a function of voltage, the net retardation as a function of wavelength can be extracted. We perform a Jones matrix analysis [9] in which the polarization state of the light is tracked through the optical system via matrix elements that represent each of the three optical elements shown in Fig. 2(b). In this way, the retardation of the liquid-crystal cell can be linked to the measured attenuation of the SLM system.

Consider the normalized retardation residual, given by

\[
\Delta \rho(\lambda, V) = \frac{\rho(\lambda, V) - \langle \rho(\lambda, V) \rangle_{\lambda}}{\langle \rho(\lambda, V) \rangle_{\lambda}},
\]

where \(\rho(\lambda, V)\) is the net retardation as a function of wavelength and voltage, and the brackets indicate averaging over wavelength. Given a large enough spectral range to average over, the normalized retardation residual should show variation as a function of voltage in the presence of secondary reflections.
of secondary reflection phase effects since it is governed in part by the birefringence of the cell.

Figure 4 shows the normalized retardation residual for the liquid-crystal cell used in Fig. 3. The averaging was done over a 1525-1565 nm wavelength band in 0.5 nm increments. The traces in Fig. 4 clearly show signs of secondary reflection effects. As the retardation of the cell is changed with voltage, the secondary retardation is also changed, resulting in the crossing of the curves shown in Fig. 4.

This behavior can be further confirmed by numerically modeling the liquid-crystal cell. We use a simple interface-and-propagation matrix method [11], wherein a 2x2 matrix is used to describe the forward and backward propagating fields through each layer and at each boundary. In applying this method, the liquid crystal is treated as a series of thin films, each with constant refractive index for the ordinary and extraordinary polarizations. These indices are obtained from the orientation of the liquid-crystal molecules in each thin film, which are calculated by balancing elastic and electrostatic energies in the total liquid-crystal layer [7]. For the PAN cell configuration used, only the molecular tilt has significant impact to the birefringence. The matrix describing the propagation through the entire cell is obtained by multiplying the interface and propagation matrices together in the appropriate order. The reflected optical fields for the ordinary and extraordinary polarizations are then calculated separately by applying the appropriate polarization vector, and the phase difference is obtained from the resultant reflected field values.

Fig. 4. Normalized retardation residuals of liquid-crystal cell as a function of voltage for various wavelengths.
The normalized retardation residual obtained through these calculations are shown in Fig. 5, using the bandwidth and wavelength spacing identical to those used for the experimental case. The data is qualitatively identical to that shown in Fig. 3 in that the normalized retardation residuals change as a function of voltage and wavelength and also exhibit the crossover phenomenon. The difference in the quantitative magnitude of the normalized retardation residual compared to the experimental values is likely due to slight differences in the fabricated device compared to the design (i.e., modeled device). For example, our simulations have shown +/-5% variability in the thickness of the liquid-crystal layer can lead to a 3.5% swing in the residual retardation.

By setting all Fresnel reflection and transmission coefficients to zero and unity, respectively, with the exception of the gold reflector, the liquid-crystal cell can be modeled without any secondary reflections. These simulations show that in the absence of secondary reflections, the normalized retardation residuals are constant as a function of voltage. The value of the constant is solely due to the incident wavelength. Therefore, the data presented in Fig. 4 proves the existence of residual retardation due to secondary reflection effects.

4. Discussion and conclusions

For the particular liquid-crystal cell geometry used in these experiments, a change in the expected retardation on the order of +/- 3% can be expected, and perhaps as large as +/- 10% for alternative geometries. The magnitude and sign of this change is a function of wavelength and voltage, leading to several interesting ramifications.
First, consider the performance effects of secondary reflections on amplitude modulation through an SLM system whose pixels would be described by Fig. 2(b). An extension of Malus’ law [9] to imperfect polarizers can be derived from Jones matrix analysis and is approximately given by

\[
P_{\text{out}} = P_{\text{in}} \left[ \cos^2 \left( \frac{\phi}{2} \right) + \varepsilon^2 \sin^2 \left( \frac{\phi}{2} \right) \right] \tag{8}
\]

where \( \phi \) is the retardation in radians (or equivalently, twice the rotation angle of the polarization with respect to the polarizer), \( \varepsilon^2 \) is the polarizer extinction, \( P_{\text{in}} \) is the power incident on the polarizer, and \( P_{\text{out}} \) is the power that passes through the polarizer. Equation (8) is valid provided \( \varepsilon \) is much less than unity.

Any change in the expected retardation will change the expected rotation angle of the polarized light with respect to the polarizer. Thus, any amount of residual retardation caused by secondary reflections will change the system attenuation. Figure 6 shows the attenuation for several levels of residual retardation as a function of the expected attenuation, the case for which there are no secondary reflections. The polarizer extinction is assumed to be -40 dB. From this plot, it is clear that there can be a significant change in the performance of the SLM due to secondary reflections. In terms of maximum extinction, the change can as large as 20 dB for the current configuration, but perhaps as large as 30 dB for other cell geometries. For attenuation on the order of 5-10 dB, for example in beam shaping applications, the change can be as large as 10-20% of the proper attenuation. This magnitude of error will certainly lead to a beam shape that deviates from that which is desired.
In practice, the voltage vs. attenuation curves are determined experimentally for each cell at a given wavelength. Thus, the difference between expected vs. actual behavior may not even be noticed. However, depending on the spectral properties of the incident light, achieving the maximum expected extinction may not be feasible. The effect of the incident wavelength is embedded in both the nominal birefringence and the linear phase accrual through the cell, both of which play an important role in determining the magnitude of the residual retardation. Even for small attenuation levels used in beam shaping applications, the changes that occur due to a broad-band incident beam may also lead to severe departures from the expected beam shape. The desired beam shape may not even be achievable, depending on the bandwidth of the incident beam. Figure 6 is still instructive in understanding the behavior of broad-band incident light, if one considers the different spectral components to have differing values of normalized retardation residuals.

In the fabrication of liquid-crystal cells, variability in the process can lead to variability in the nominal performance of the cell. The inclusion of secondary reflections exacerbates this problem. Additionally, if there are any changes in the liquid-crystal cell over the course of its lifetime, for example swelling of a polymer layer due to moisture absorption, the behavior of the device will change due to secondary reflections through the isotropic phase accrual, as given in Eq. (5).

Another issue arising from secondary reflections that may be of importance is when the incident light is pulsed. For example, a 5-um thick cell will lead to secondary pulses approximately 50-fs ahead and behind the main pulse. While this may not necessarily be an issue for many applications, consider pulses from an ultra-fast laser [12]. In this case, the leading and trailing pulses that are of similar duration to the original pulse may be catastrophic to obtaining the desired results.

It should be noted that for a transmissive liquid-crystal cell, this reflection problem is significantly reduced since the coherently added beam must experience two weak reflections. This is approximately equivalent to squaring the fractional normalized retardation residual. However, depending on the configuration and composition of the layers in the cell, secondary reflections may play an important role in determining the operational quality of the device [6].

In conclusion, the existence of secondary reflections in reflective liquid-crystal cells have been predicted and verified. The wavelength dependence of the phenomena was used to explore the magnitude of the effect. Changes of the retardation due to these secondary reflections were measured to be +/- 3% for the particular cell under study. Numerical modeling verified the root cause of these changes to be secondary reflections, and predicted that as much as 10% change can be obtained for different liquid-crystal cell thicknesses and material compositions. The deleterious effects of these secondary reflections were explored from a device performance perspective, and found to be most harmful to system performance for incident light with a broad spectral bandwidth.

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