Research in Quantum Information

Tom Cover
(Stanford MURI Contract DAAD-19-99-1-0215)
2/13/02

- A statistical view of Bell's inequality
  (T. Cover, in preparation)

- The amount of information in a quantum oracle.
  (T. Cover, ongoing)

- Duality between channel capacity and data compression with state information.
  (T. Cover, and M. Chiang, to appear IEEE Trans. Info. Theory)

- Quantum Slepian Wolf theorem.
  (Jon Yard, ongoing)

- Quantum multiple access capacity
  (Gleb Klimovitch, ongoing)

- Source capacity over a source dependent channel.
  (Cover, Sutivong, Chiang, ongoing)

- Worst additive noise
  (Cover and Diggavi, ISIT November 2001)

- Concavity of the second law of thermodynamics.
  (T. Cover, D. Julian, ongoing)

Researchers: T. Cover, J. Yard, D. Julian

How does physics change information theory?

- geometry: relativity

What is information theory?

- Asymptotic equipartition property (AEP)
  "almost everything is equally probable"

$$p(x^n) \doteq 2^{-nH}$$

- Channel capacity
  (Distinguishability)

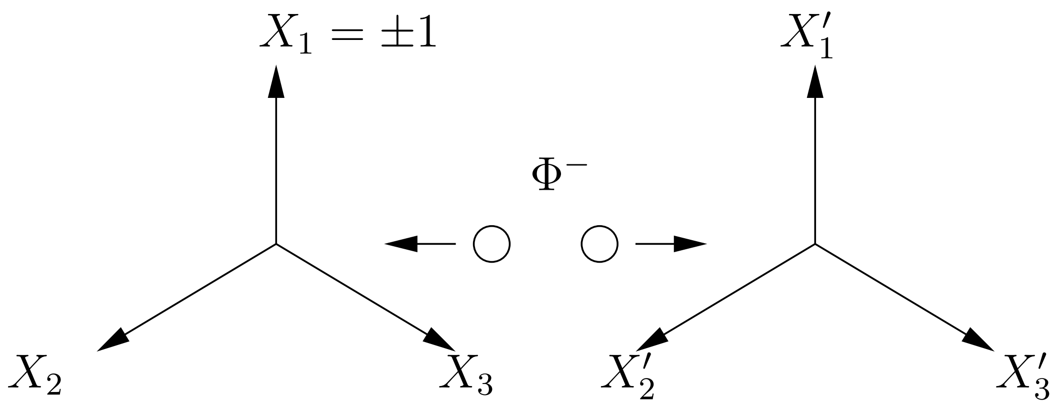
$$C = \max I(X;Y)$$

- Entropy $H = -\sum p \log p$
  (Descriptive complexity)

$$K(x)$$

- Information theory: QM

A view of Bell's Inequality



$$\Phi^- = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

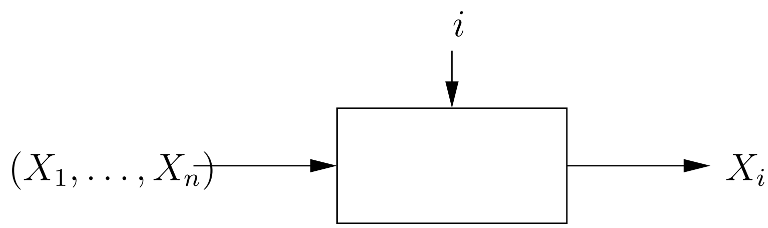
$$\begin{aligned} 1 &\leq \mathbf{E}(X_1 + X_2 + X_3)^2 \\ &= \mathbf{E}X_1^2 + \mathbf{E}X_2^2 + \mathbf{E}X_3^2 + 2\mathbf{E}X_1X_2 + 2\mathbf{E}X_1X_3 + 2\mathbf{E}X_2X_3 \\ &= 1 + 1 + 1 - 1 - 1 - 1 \\ &= 0 \end{aligned}$$

How much information in a qubit oracle?

- Classical bit: Store one bit

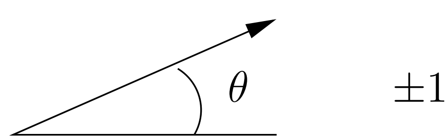
$$\boxed{1}$$

- Classical oracle: Answer to any one bit question about library $X_1, \ldots, X_n$.



- Must store $\mathrm{H}(X_1, X_2, \ldots, X_n)$ bits in oracle

- Quantum:
  — What will be result of polarization measurement at angle θ?



- Infinite number of classical bits will not suffice.

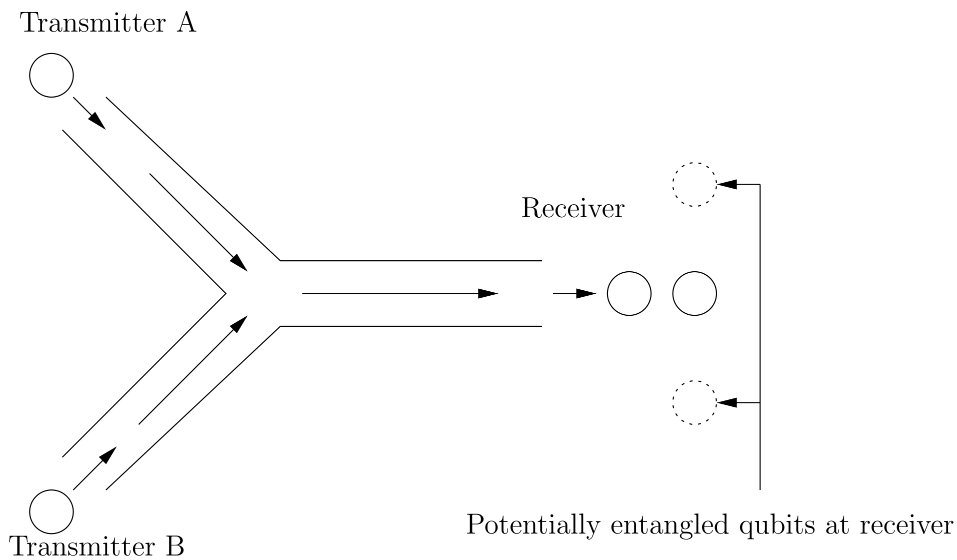
$$\Psi = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

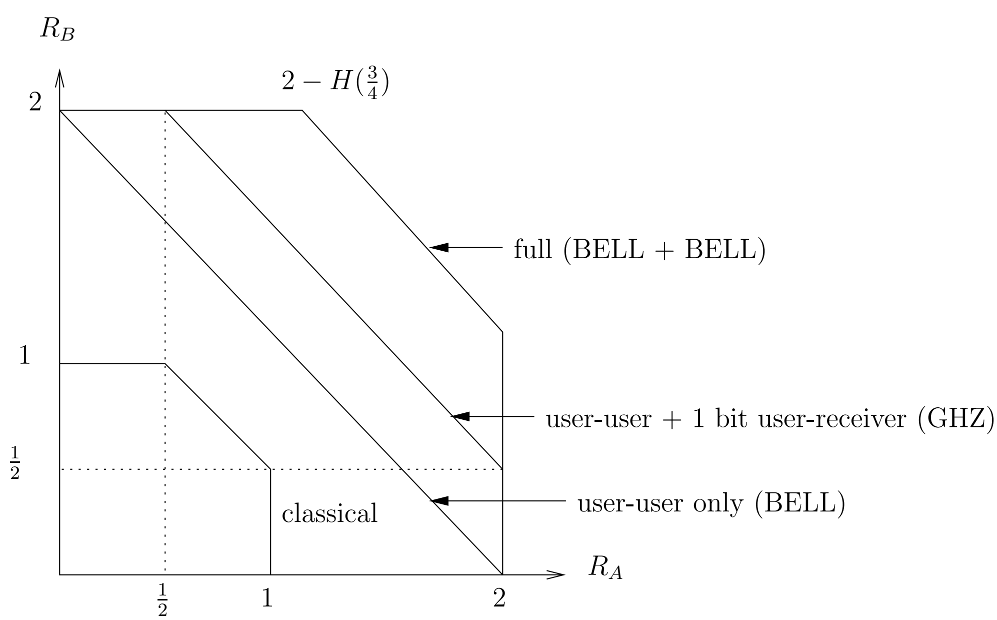
- One entangled qubit suffices.

Quantum multiple access channel Capacity

G. Klimovitch and A. Winter

- Quantum Adder Channel





- 1-bit entanglement (GHZ setup)

$$\Psi = \frac{1}{\sqrt{2}} \left(|00\rangle_{AB} |\alpha\rangle_R + |11\rangle_{AB} |\beta\rangle_R \right)$$

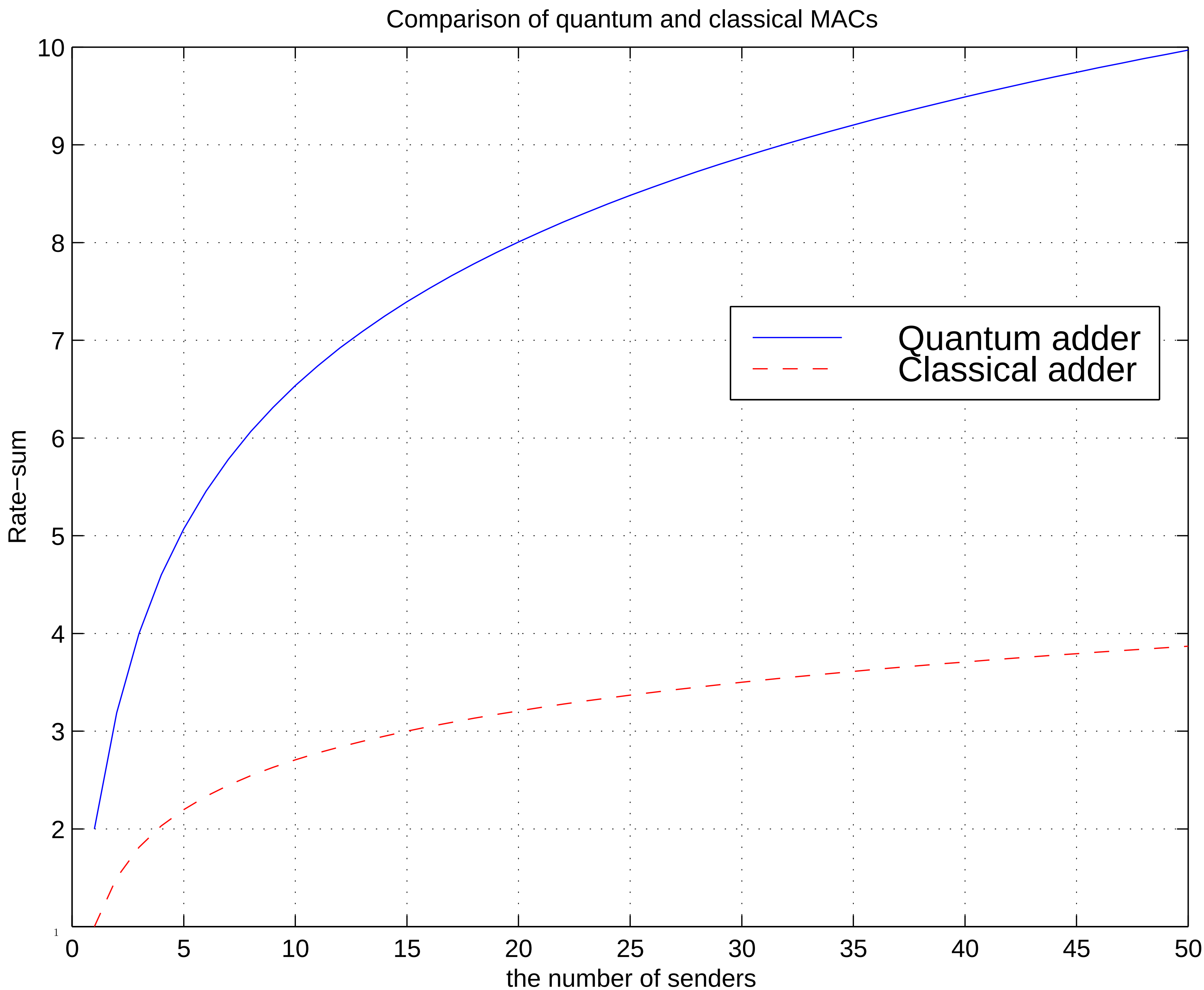
- Full entanglement (Bell + Bell setup)

$$\Psi = \frac{1}{2} \left(|00\rangle_{AB} |\alpha\rangle_R + |01\rangle_{AB} |\beta\rangle_R + |10\rangle_{AB} |\gamma\rangle_R + |11\rangle_{AB} |\delta\rangle_R \right)$$

- User-user entanglement only is similar to superdense coding, but

  - it is distributed coding

  - the two sent qubits are indistinguishable at receiver

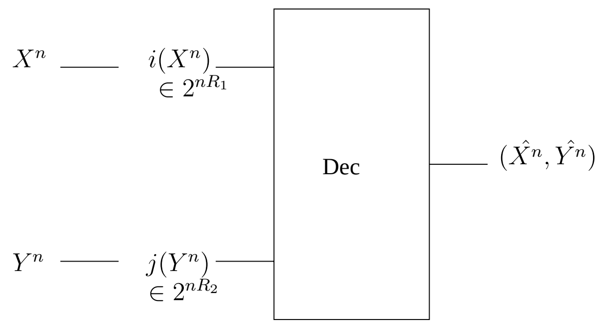

Comparison of quantum and classical MACs
Quantum adder
Classical adder
Rate−sum
the number of senders
1
2
3
4
5
6
7
8
9
10
0
5
10
15
20
25
30
35
40
45
50

Quantum Slepian-Wolf

(Jon Yard)

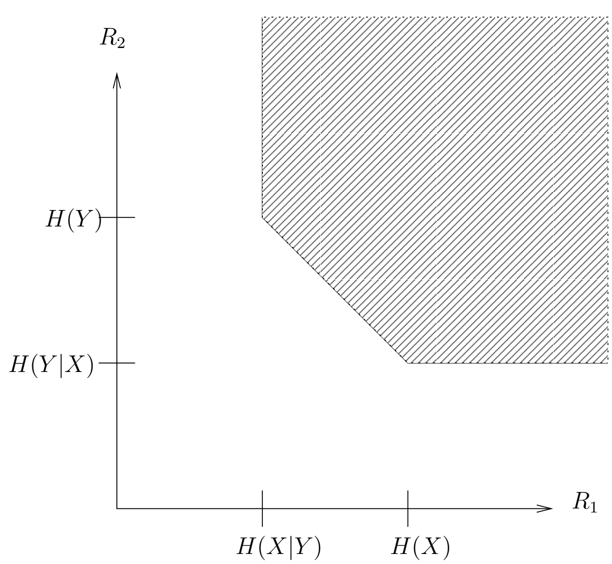
- Classical:

$$(X_i, Y_i) \sim p(x,y)$$

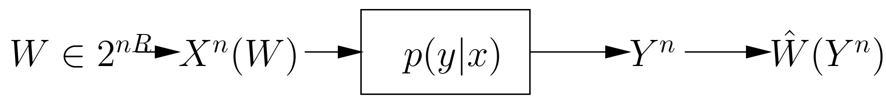


- Theorem:

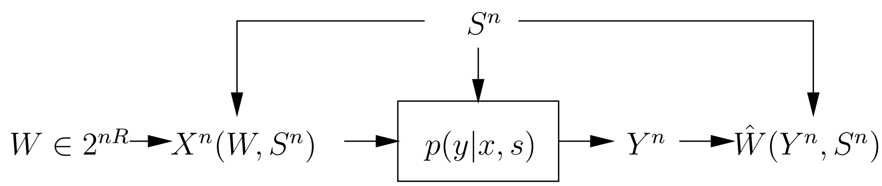
$$\begin{aligned} R_1 &\geq H(X|Y) \\ R_2 &\geq H(Y|X) \\ R_1 + R_2 &\geq H(X,Y) \end{aligned}$$



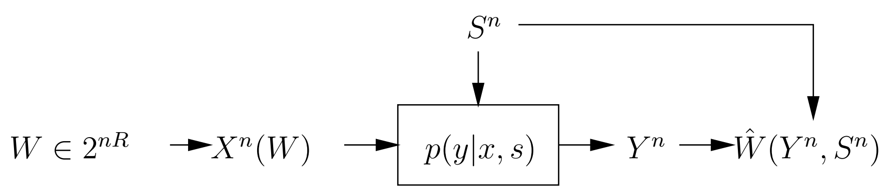
Channel capacity with side information



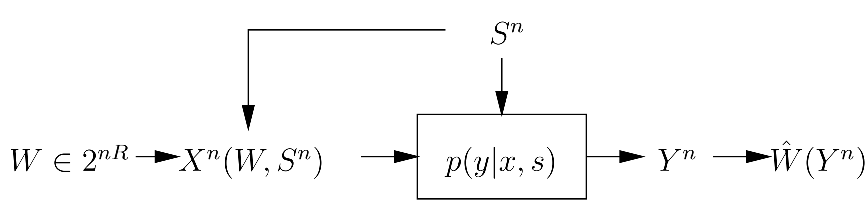
$$C_{00} = \max_{p(x)} I(X;Y)$$



$$C_{11} = \max_{p(x|s)} I(X;Y|S)$$



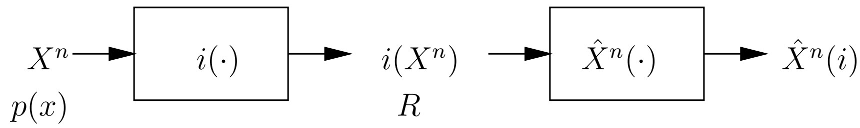
$$C_{01} = \max_{p(x)} I(X;Y|S)$$



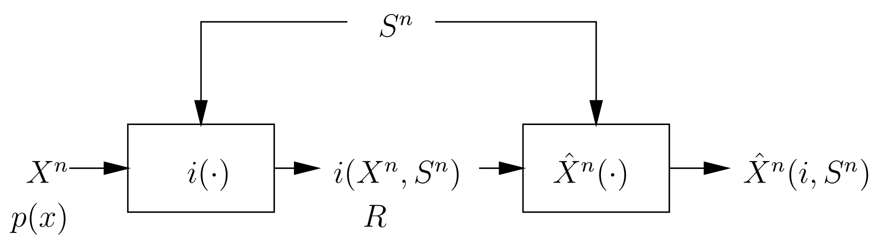
$$C_{10} = \max_{p(u,x|s)} (I(U;Y) - I(U;S))$$

Gel'fand and Pinsker

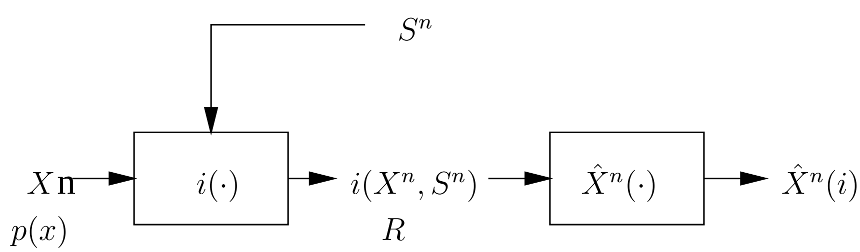
Rate-distortion with side information



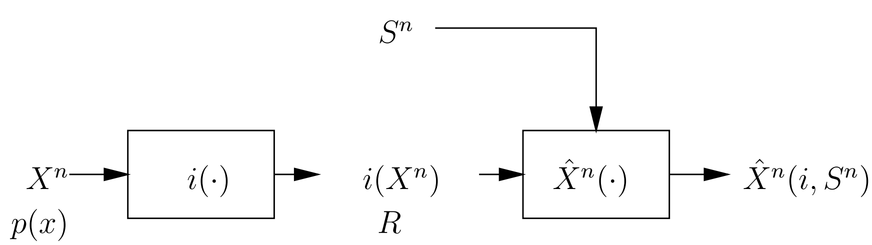
$$R_{00} = \min_{p(\hat{x}|x):Ed(X,\hat{X})\leq D} I(X;\hat{X})$$



$$R_{11} = \min_{p(\hat{x}|x,s)} I(X;\hat{X}|S)$$



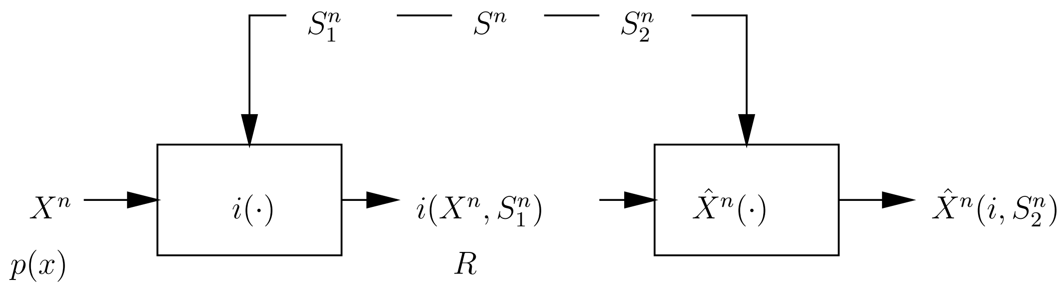
$$R_{10} = \min_{p(\hat{x}|x)} I(X;\hat{X})$$



$$R_{01} = \max_{p(u|x)p(\hat{x}|u,s)} (I(U;X) - I(U;S))$$

Wyner and Ziv

Generalized rate-distortion with side information



Source: $(X_i, S_{1i}, S_{2i}) \sim$ i.i.d. $p(x, s_1, s_2)$
Distortion: $d(x, \hat{x})$

$$R(D) = \min_{p(u|x,s_1)p(\hat{x}|u,s_2):Ed(X,\hat{X})\le D} (I(U;X,S_1) - I(U;S_2))$$

Duality

• Channel capacity

$$C_{S_1S_2} = \max_{p(u|s_1)p(x|u,s_1)} (I(U;Y,S_2) - I(U;S_1))$$

•Rate-distortion function

$$R_{S_1S_2}(D) = \min_{p(u|x,s_1)p(\hat{x}|u,s_2):Ed(X,\hat{X})\le D} (I(U;X,S_1) - I(U;S_2))$$

Second Law

- Does entropy increase?
- Consider stationary Markov chains:

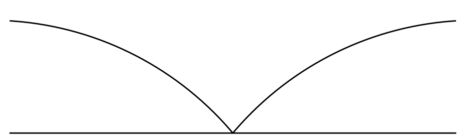
$$P = [P_{ij}]$$

- Stationary distribution μ:

$$\mu_j = \sum \mu_i P_{ij}$$

- Let $\{X_n\}$ be a stationary Markov chain.

1. $\mathsf{H}(X_n) = -\sum_{ij} \mu_i P_{ij} \log P_{ij}$
   Entropy is constant.
2. $\mathsf{H}(X_n|X_0) \nearrow$
   Conditional entropy increases.
3. $\mathsf{H}(X_{-n}|X_0) = \mathsf{H}(X_n|X_0)$
   Symmetry.



4. (with D. Julian)
   $\mathsf{H}(X_n|X_0)$ is concave.

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