Gate-Controlled Spin-Orbit Quantum Interference Effects in Lateral Transport


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In situ control of spin-orbit coupling in coherent transport using a clean GaAs/AlGaAs two-dimensional electron gas is realized, leading to a gate-tunable crossover from weak localization to antilocalization. The necessary theory of 2D magnetotransport in the presence of spin-orbit coupling beyond the diffusive approximation is developed and used to analyze experimental data. With this theory the Rashba contribution and linear and cubic Dresselhaus contributions to spin-orbit coupling are separately estimated, allowing the angular dependence of spin-orbit precession to be extracted at various gate voltages.

An important component along the path toward realizing quantum “spintronic” devices [1,2] is a structure that allows manipulation of electron spin without destroying phase coherence. Spin-orbit (SO) coupling has been the focus of recent studies because of its potentially useful role in coherent spin rotators [3], spin interference devices [4], and spin filters [5,6]. The mechanisms by which SO coupling affects transport [7–10] have recently been considered in the context of Aharonov-Bohm (AB) phase and Berry phase [4,11–16], underscoring the richness of the underlying physics.

The conductivity of low-dimensional systems shows signatures of quantum interference that depend on magnetic field and SO coupling [7,8,17–19]. In particular, constructive (destructive) backscattering associated with pairs of time-reversed closed-loop electron trajectories in the absence (presence) of significant SO interaction leads to negative (positive) magnetoresistance effects known as weak localization (antilocalization) [9].

In this Letter, we demonstrate in situ control of SO coupling in a moderately high-mobility GaAs/AlGaAs two-dimensional electron gas (2DEG), inducing a crossover from weak localization (WL) to antilocalization (AL) as a function of an applied top-gate voltage (see Fig. 1). Theory beyond the diffusive approximation must be used to extract gate-voltage-dependent SO parameters from magnetotransport when the SO precession frequency becomes comparable to the inverse transport scattering time ($\tau^{-1}$) as occurs here, and when the magnetic length becomes comparable to the mean free path. Theory that accounts for AB-like spin phases and spin relaxation [20] is developed here and used to estimate separately the various SO terms (Rashba, linear, and cubic Dresselhaus) over a range of gate voltages, ranging from WL to AL.

Conventional WL theories assume SO times much longer than $\tau$ [7,8,13] and so cannot be applied to clean materials. Previous theories that go beyond the diffusive approximation do not treat SO [21,22], or treat it only as spin relaxation [23,24] without accounting for Berry phase effects which play a crucial role, as we show here.

![Graph](image_url)

FIG. 1. (a) Experimental magnetoconductance, $\Delta \sigma = \sigma(B) - \sigma(0)$ (circles), offset for clarity, along with three-parameter fits to Eq. (2) (solid curves) for several gate voltages. Inset: Experimental magnetoconductance data for the most negative gate voltage, showing pure WL. (b) Density and mobility as a function of $V_g$, extracted from longitudinal and Hall voltage measurements. (c) Experimental conductivity, showing strong dependence on $V_g$. Note that $\Delta \sigma \sim 10^3 \sigma$. 

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Previous experiments in which SO rates are measured using WL/AL in a gated GaAs heterostructure have not reported in situ gate control [10, 25, 26]. Koga et al. [27] demonstrated gate controlled SO coupling in InGaAs heterostructures using WL/AL. Modification of Rashba SO coupling using gated quantum wells has been observed using beating patterns in Shubnikov–de Haas oscillations in InGaAs [28, 29], InAs/AlSb [30] and HgTe [31]. Gate controlled SO coupling in GaAs 2D hole systems [32–34] has also been investigated using beating of Shubnikov–de Haas oscillations. The angular variation of SO coupling in GaAs quantum wells has been measured using Raman scattering [35].

The Hamiltonian for conduction band electrons in a [001] 2DEG is $H = \frac{\hbar^2 k^2}{2m} + (\mathbf{\sigma} \cdot \mathbf{A})$, where $m^*$ is the effective mass, $k = |\mathbf{k}| \ [k = (k_x, k_y)]$ is the in-plane wave vector, $\mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli spin operator, and $\mathbf{A} = (\mathbf{A}_x, \mathbf{A}_y)$ is the total SO frequency. $\mathbf{A}$ can be written as the vector sum of linear ($\mathbf{A}_{D1}$) and cubic ($\mathbf{A}_{D3}$) Dresselhaus terms and the Rashba term ($\mathbf{A}_{R}$):

$$\mathbf{A}_{D1} = \alpha_1 (-\hat{x} k_x + \hat{y} k_y) / \hbar, \quad (1a)$$
$$\mathbf{A}_{R} = \alpha_2 (\hat{x} k_y - \hat{y} k_x) / \hbar, \quad (1b)$$
$$\mathbf{A}_{D3} = \gamma (\hat{x} k_x^2 - \hat{y} k_y^2) / \hbar. \quad (1c)$$

where $\gamma$ arises from the lack of inversion symmetry of the GaAs crystal, while $\alpha_1 = \gamma k_z^2$ also depends on the thickness of the wave function in the quantization direction. $\alpha_2$ depends on the potential profile of the heterointerface. We assume the effect of gate voltage, $V_g$, on $\mathbf{A}$ ($\equiv |\mathbf{A}|$) is through the carrier density, $n = k^2 / 2\pi$. Previous studies of SO coupling in single-interface heterostructures [36] support this assumption. The magnitude of $\alpha_2$ in a single-interface heterostructure originates mainly from the band offset at the heterointerface, which is roughly independent of $V_g$ [30, 37].

The symmetry of the linear (in $k$) SO terms, $\mathbf{A}_{D1}$ and $\mathbf{A}_{R}$, allows these terms to be represented as a spin-dependent vector potential $\mathbf{A}$ that affects the orbital motion and phase of electrons, $\mathbf{\sigma} \cdot (\mathbf{A}_{D1} + \mathbf{A}_{R}) \approx \mathbf{k} \cdot \mathbf{A}$ [4,11–15]. That is, the linear terms affect electronic interference as a spin-dependent AB-like effect. In contrast, the cubic term, Eq. (1c), upon removing terms with the symmetry of Eq. (1a), causes only spin relaxation in the diffusive regime (although it also can produce AB-like effects in the quasiballistic regime [4]).

To develop the theory of 2D magnetotransport with SO coupling beyond the diffusive approximation [20], we follow Refs. [21–23], which treat the quasiballistic case $\ell_B < \ell$ ($\ell_B = \sqrt{\hbar / 2eB}$ is the magnetic length and $\ell$ is the transport mean free path) without SO coupling. The approach is to introduce an operator $P = G^{G^*}_{\mathbf{r}_1 \mathbf{r}_2} (\mathbf{r}_1, \mathbf{r}_2, \mathbf{\sigma}) G^2 (\mathbf{r}_1, \mathbf{r}_2, \mathbf{\sigma}) \hbar / 2\pi \tau$ for the probability of an electron to propagate both forward and backward along a path segment from $\mathbf{r}_1$ to $\mathbf{r}_2$, where $G^R (G^A)$ are single-electron retarded (advanced) Green functions, $\mathbf{\sigma}_{(2)}$ are the Pauli spin operators for particles moving forward (backward), $\nu$ is the density of states per spin. The interference contribution from the $n$th traversal of a closed path is given by the trace of $(P)^n$. In the presence of SO coupling, Eq. (1), the formulas in [22] remain valid once a summation over spins is included in the trace.

Introducing the total spin of interfering particle waves, $S = \mathbf{\sigma}_1 + \mathbf{\sigma}_2$, we write $\text{Tr}((P)^n) = \frac{1}{2} \text{Tr}((P^1)^n) - (P_0^n)$, where operators $P_0$ and $P_1$ describe singlet ($S = 0$) and triplet ($S = 1$) contributions. To calculate $\text{Tr}((P_0^{(1)})^n)$, we diagonalize $P_0([1])$. We find that when $\mathbf{A}_{D1}$ and $\mathbf{A}_{R}$ are taken into account, $P_0([1])$ has the same eigenfunctions as the Hamiltonian $\mathbf{H}$ for particles with charge $2e, \mathbf{S}$, and spin frequency $2\Omega$: $\mathbf{H} = \frac{\hbar^2}{2m}(2 - 2e\mathbf{A}_m + 2e\mathbf{A}_s)^2$, where $\mathbf{A}_m$ is the vector potential associated with the applied perpendicular magnetic field, $B$, and $\mathbf{A}_s = \frac{m_e}{2e}(\alpha_1 \mathbf{S} - \alpha_2 \mathbf{S} \mathbf{S})/eB$ is the SO vector potential. For $S = 0$, the eigenstates are Landau levels for a charge $2e$ particle in the magnetic field $B$, analogous to the spinless problem [23]. For $S = 1$, eigenstates of $\mathbf{H}$ and $P_1$ in general require a numerical solution, although analytic solutions exist when either $\alpha_1$ or $\alpha_2$ equals zero [20]. An analytic solution is also found for $\alpha_1$, $\alpha_2 \neq 0$, when $\ell_B < \ell_{so}$, where $\ell_{so} = (2\alpha_1 m^*/\hbar^2)^{-1}$ is the distance over which spin rotates appreciably (if $\ell < \ell_{so}$) or dephases (if $\ell < \ell_{so}$) due to spin AB-like effects. Performing a unitary transformation $\mathbf{H} \rightarrow \mathbf{H}' = U^\dag \mathbf{H} U$, with $U = \exp(-i\mathbf{A}_s \cdot \mathbf{r})$, and expanding in coordinates, we find $\mathbf{H}' = \frac{\hbar^2}{2m} (k - 2e\mathbf{A}_m + 2e\mathbf{A}_s)^2$, where $\mathbf{a} = \mathbf{H}_0 \mathbf{r} \times \mathbf{A} / (2B^2)$, and $\mathbf{H}_0 = (2\alpha_1^2 - \alpha_1^2 m^2/\hbar^2)$ is the effective SO field. $P_1$ can then be block diagonalized for each $m (m = 0, \pm 1)$ using the Landau basis for particles with charge $2e$ in the magnetic field $B = m\mathbf{H}_0$. Thus, the effect of $\mathbf{A}_{D1}$ and $\mathbf{A}_{R}$ is to produce AB-like spin quantal phases [4,11–15]. Higher expansion terms to $\mathbf{H}'$ describe spin flip processes and can be taken into account by introducing a spin-relaxation time $T_{so}$ and its corresponding field scale $H_{so} = \hbar T_s / (2eB^2 T_{so})$. The resulting quantum interference contribution takes the form [20]

$$\Delta \sigma(B) = \frac{-\hbar^2 e^2}{4\pi^2 \hbar} \left[ \sum_{m=-1,0,1} C(x_{1m}, f_{1m}) - C(x_{00}, f_{00}) \right],$$

where $x_{Sm} = (B - m\mathbf{H}_0) / \mathbf{H}_0$ describes the AB dephasing in $H_{so}$. $C(x, f_{sm}) = x \sum_{n=0} x P_n(x_{sm})$, $P_n(x_{sm}) = \gamma \int \exp(-y^2 f_{sm} - t^2 / 2L_n(t^2) dt, L_n(t^2))$ are Laguerre polynomials, $y = (2 / |x|)^{1/2}$. and $\mathbf{H}_0 = \hbar T_s / (2eB^{2})$. The dephasing factors $f_{sm}$ are given by $f_{z1} = [1 + (H_\phi + H_{so}) / H_0]^2$, $f_{z0} = (1 + H_\phi / H_0)$, $f_{z0} = (1 + (H_\phi + 2H_{so}) / H_0)$, where $H_\phi = \hbar / (4eL_0^2)$ and $L_0$ is the phase breaking length.

Equation (2) does not include all $B$-dependent interference terms, notably excluding Cooper-channel contributions due to electron-electron interactions [18] and a
ductance as a function of currents ranging from 50 to 500 nA. 

reduction of WL due to electron diffraction effects [21]. Also, in an attempt to capture the effects of cubic terms on $H_{\text{eff}}$ and $H_{\text{so}}$, we introduce an effective vector potential $A_{\text{eff}} = A_x + \gamma \frac{m}{2\pi} (k_x^2 - k_y^2) \sim A_x + \gamma \frac{m}{2\pi} (k_x^2 - k_y^2)$ which leads to an effective SO field,

$$H_{\text{so}}^* = 2(\alpha_2^2 - \alpha_1^2 + 2\pi n\alpha_1\gamma - \pi^2\gamma^2 n^2)2^m2^e/\hbar^3. \quad (3)$$

Equation (2) is applicable when $B > H_{\text{eff}}^*$ (see Fig. 2). We have confirmed that fitting only to data where $B > H_{\text{eff}}^*$ gives, within error bars, the same results as fitting over the entire measured range of $B$.

Modification of the commutator $[k + 2eA_{\text{eff}}^* \mathbf{r}]$ by $A_{\text{eff}}^*$ induces spin flipping terms $\sim k^3/4$ in the transformed Hamiltonian $\mathbf{H}^*$. The corresponding $H_{\text{eff}}^* = \frac{\partial}{\partial \mathbf{k}} \pi^2\gamma^2 n^2/\hbar^3$, using its expression in the diffusive regime.

We now turn to a discussion of the experiment. Samples on three separate heterostructure materials all showed qualitatively similar behavior. The sample for which data are presented consists of a GaAs/AlGaAs heterostructure grown in the [001] direction with double $\delta$-doping layers set back 143 and 161 A˚ from the 2DEG and a total distance of 349 A˚ from the surface to the 2DEG. A 200 $\mu$m wide Hall bar with 700 $\mu$m between voltage probes was patterned by wet etching. A lithographically defined Cr/Au top gate was used to control density and mobility in the Hall bar over the range $n = (1.4–7.0) \times 10^{15}$ m$^{-2}$ and $\mu = 3.6–31$ m$^2$/Vs. Measurements were made in a $^3$He cryostat at temperature $T = 300$ mK using ac lock-in techniques with bias currents ranging from 50 to 500 nA.

Figure 1(a) shows the longitudinal magnetoconductance as a function of $V_g$. A crossover from pure WL [Fig. 1(a), inset] at $V_g = -240$ mV to essentially pure AL at $V_g = +250$ mV is observed. This crossover demonstrates that a gate can be used to control SO over a wide range, as pure WL corresponds to negligible SO rotations within the phase coherence length $L_g$, while AL corresponds to spin rotations $\geq 2\pi$. The solid curves in Fig. 1(a) are fits of Eq. (2) with three free parameters, $H_{\text{eff}}$, $H_{\text{so}}$, and $H_{\text{eff}}'$. $H_{\text{eff}}$ is fixed at each gate voltage by measured values of density and mobility.

Figure 2 shows extracted parameters $H_{\text{so}}^*$ and $H_{\text{eff}}^*$ as a function of $n^2$. $H_{\text{so}}^*$ is well described by the predicted linear dependence on $n^2$, with a best fit (Fig. 2, solid line) giving $\gamma = 31 \pm 3$ eV A$^3$ with zero $\gamma$ intercept [see Eq. (1c)]. The density dependence of $H_{\text{eff}}^*$ is well described by Eq. (3), (Fig. 2, dotted curve), giving fit parameters $\gamma = 28 \pm 4$ eV A$^3$, $\alpha_1 = 4 \pm 1$ meV A, and $\alpha_2 = 5 \pm 1$ meV A. In this way, the three SO parameters $\alpha_1$, $\alpha_2$, and $\gamma$ are separately obtained from transport measurements by explicitly making use of the density dependence of $H_{\text{eff}}^*$ and $H_{\text{so}}^*$. Extracted values of $H_{\text{eff}}'$ correspond to dephasing times in the range $\tau_{\text{eff}} \sim 0.1–1.0$ ns at 300 mK, which decrease by more than an order of magnitude as temperature is increased to 2.5 K. Within the error bars, $H_{\text{so}}$ and $H_{\text{eff}}'$ do not depend on temperature over this temperature range.

Figure 3(a) displays the magnitudes of the three spin-orbit terms as functions of $V_g$, $n$, and $\mu$, determined using Eq. (1) and the extracted values of $\alpha_1$, $\alpha_2$, and $\gamma$. Plotted are values along the [110] direction, $\phi = \tan^{-1}(k_y/k_x) = \frac{\pi}{4}$, where $\Omega_{D1}$ is maximum.

The total spin precession rate, $\Omega$, is plotted as a function of the direction, $\phi$, of the electron momentum in Fig. 3(b). While for most directions $\Omega$ is an increasing function of density, it is seen to decrease with increasing density near $\phi = \frac{\pi}{4}$ and $\frac{3\pi}{4}$. The linear Dresselhaus and Rashba terms ($\Omega_{D1}$ and $\Omega_R$) are of comparable magnitude to each other for all densities and in all directions. Near $\phi = \frac{\pi}{4}$ (an integer), $\Omega_{D1} \ll \Omega_{D1}$, $\Omega_R$, and the SO is controlled by the linear terms. For $\phi$ near $\frac{\pi}{2}(j + 1/4)$, $\phi = \frac{\pi}{2}(j + 1/4)$, the cubic term becomes comparable to or even exceeds (at high densities) the linear terms. Depending on $\phi$, the linear and cubic terms either add ($\phi \sim \frac{\pi}{4}, \frac{3\pi}{4}$) or subtract ($\phi \sim \frac{\pi}{4}, \frac{3\pi}{4}$).

The extracted values for $\gamma (31 \pm 3$ eV A$^3$ using $H_{\text{so}}^*$, $28 \pm 4$ eV A$^3$ using $H_{\text{eff}}^*$) are in good agreement with the value $27.5$ eV A$^3$ from band structure calculations [36,37]. Values of $\Omega$ are $\sim 3–8$ times smaller than previously measured using Shubnikov–de Haas oscillations [26], with corresponding theory [26,37] lying roughly between the experimental ranges. We note, however, that the values are sample dependent. Estimates for $\alpha_1$ give values for $\langle k_x^2 \rangle$ that correspond to a wave function width of $\sim 10$ nm in the $\hat{x}$ direction, which is also reasonable. The extracted $\alpha_2$ corresponds to a uniform electric field $E \sim 10$ MV/m, using $\alpha_2 = \alpha_2 p E$ and a value of $\alpha_0 = 5.33 \AA^2$ from a $k \cdot p$ model [36,37].

Previously existing models for WL/AL [8,13,36] provide fits to the data that appear qualitatively reasonable,
unphysical result that was found using Eq. (2). However, these fits also lead to the conclusion that a theory for arbitrarily strong SO coupling [14] may be used to fit this data by including a term [when $\phi = (j + \frac{1}{2})\pi$] is shown for the anisotropic ($\Omega_{D1}$) term. In insets show theoretical dependence on momentum direction for the three terms, indicating that the linear terms are isotropic, while the cubic term has a fourfold symmetry and is highly anisotropic. Maximum magnitude of $\Omega$ is shown for the anisotropic ($\Omega_{D1}$) term. (b) Angular variation of $\Omega$, the magnitude of the total SO precession vector at $V_g = -150$ mV (dotted), 0 mV (dashed), and 250 mV (solid), corresponding to densities of $2.3 \times 10^{15}$, $5.0 \times 10^{15}$, and $7.0 \times 10^{15}$ m$^{-2}$, respectively.

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