Engineering the Indistinguishability and Entanglement of Two Photons

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We present a novel interferometric technique for suppressing distinguishing information in the space-time component of the state vector of an entangled pair of photons by providing two indistinguishable ways for each photon to occupy any given space-time mode. We demonstrate this method by using spontaneous parametric down-conversion to generate a pair of photons in the state with the least distinguishing information consistent with the set of modes available. The technique also allows the preparation of the two photons in a highly entangled space-time state provided certain criteria are met.

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Complementarity and entanglement are the most distinctive features of quantum mechanics in the sense that they lead to the starkest contrast between quantum and classical pictures of the physical world. Both are important ingredients of emerging quantum information technologies [1–3] which rely on interference. Complementarity implies that quantum interference can occur only if there exists no possible ancillary measurement which can, even in principle, distinguish between the interfering pathways [4,5]. An important issue, therefore, is how distinguishing information can be suppressed for entangled particles.

The suppression of such information is particularly crucial to the success of experiments that make use of the entanglement of only a portion of the state of two or more particles. For example, photons in entangled polarization states have been used for prototypical demonstrations of quantum teleportation [6], quantum dense coding [7], entanglement swapping [8,9], and quantum cryptography [10,11]. These experiments relied on polarization sensitive measurements, and thus it was vital to ensure that the space-time component of the state contained no distinguishing “which-path” information for the photons—such information degrades their polarization entanglement. For this reason spatial and frequency filters were used to select from the ensemble only those modes with the least distinguishing space-time information. The price for this selection, however, is that the rate of photon pair (and therefore qubit) transmission is drastically reduced.

The presence of distinguishing space-time information in several of these experiments stems from their reliance on precisely timed photons from two independent sources. Typically, these are parametric down-converters pumped by a train of ultrashort optical pulses; the pump acts as a master clock that synchronizes the emission of all of the photons in the experiment. But this timing capability comes at a price, because the broad range of frequencies available for down-conversion within the pump pulse produces which-path information by virtue of the different spectra of the two down-converted photons [12,13].

In this Letter we demonstrate a new technique that completely suppresses this spectral distinguishing information and maximizes the potential for quantum interference for the photon pairs while maintaining synchronization of the source. Consider the two-particle state,

\[
|\psi\rangle = \int \int dx dy f(x, y) |x\rangle |y\rangle,
\]

where \(|x\rangle = |1\rangle_x\) indicates a single particle in mode \(x\), and \(|y\rangle = |1\rangle_y\) indicates a particle in a distinct mode \(y\). When these particles are interfered, the paths are rendered partially distinguishable if \(f(x, y)\) is not symmetric under exchange of its arguments, because auxiliary measurements of the properties \(x\) and \(y\) can sometimes reveal which particle takes which path. On the other hand, the state

\[
|\psi\rangle_S = \int \int dx dy [f(x, y) + e^{i\theta} f(y, x)] |x\rangle |y\rangle,
\]

obtained by coherently superposing two states of the form Eq. (1), contains no such distinguishing information.

Using collinear parametric down-conversion, it is possible to generate two photons in a state of the form \(|\psi\rangle\). In this case the mode labels are frequency and polarization (although time and polarization are equally good); \(x = (\omega_x, x)\) and \(y = (\omega_x, y)\), where \(x\) and \(y\) are transverse polarization vectors for wave packets propagating in the \(z\) direction. A type-II parametric down-converter provides an amplitude function \(f(x, y)\) that contains significant distinguishing information, as revealed by means of the Hong-Ou-Mandel (HOM) interferometer.

In the HOM interferometer [14], two photons are coincident simultaneously on different input ports of a beam splitter, and four outcomes are possible: Both photons may leave the beam splitter together in one output port, or the other; both photons may be reflected into opposite ports, or both may be transmitted. The probability amplitudes of the last two outcomes have opposite phase, and cancel each other completely if the beam splitter reflects...
or transmits with equal probability. Thus, quantum interference dictates that two detectors located at the output ports of a 50/50 beam splitter will never register photons simultaneously, as long as the double-reflection and double-transmission paths to detection are indistinguishable from one another in principle [15].

When the inputs to the two ports are the signal and idler photons generated by parametric down-conversion, the joint emission spectrum may compromise this destructive interference. In particular, if a type-II down-conversion crystal is pumped by an ultrashort pulse, the bandwidths of the orthogonally polarized signal and idler photon wave packets may be quite different, owing to the dispersion characteristics of the (birefringent) down-conversion medium. This bandwidth difference is a kind of “spectral information” which renders the coincidence paths partially distinguishable; the frequencies of the detected photons could sometimes identify which photon arrived at each detector, unless the joint emission amplitude spectrum for these pairs is switched from \( f(\omega_s, \omega_i) \) to \( f(\omega_i, \omega_s) \). The phase \( \theta \) is varied by changing the position of M1 with the PZT.

The photon pairs then pass through MD1 and into the common-path HOM interferometer. With several crystal quartz plates, the \( x \)-polarized photon is delayed by an amount \( \delta \tau \) relative to the \( y \)-polarized photon. Next, a \( \lambda/2 \) plate rotates the initial \( x \) and \( y \) polarizations by 45°; Each photon may then be either reflected or transmitted at the polarizing beam splitter (PBS) with equal probability, yielding two interfering pathways for coincidence detection at a pair of EG&G SPCM-200 avalanche photodiode detectors (D_A, D_B). The signals from D_A and D_B are counted separately and in coincidence as \( \delta \tau \) is varied. Typical coincidence counting rates are 4000/sec; with the apertures \( A_p \) and \( A_B \) closed down to 2 mm and 1 mm, respectively, the coincidence rate is roughly 15/sec and each single-channel rate is about 3000/sec. These numbers imply an overall collection and detection efficiency of \( \approx 1\% \) for each detector. The apertures are necessary for interference to be observed because the modes of emission for the two creation processes are not well matched due to the different divergence properties of the pump and down-conversion beams [18]. We expect that the counting rate could be improved upon if these modes could be better matched at the source.

The experimental demonstration of this technique is depicted in Fig. 1. The output of an 80 MHz mode-locked Ti:sapphire oscillator at 810 nm is frequency doubled to give a train of 405 nm pulses with \( \approx 3 \) nm bandwidth and 330 mW average power; this pump beam is directed onto a 5-mm-long \( \beta \)-barium borate (BBO) down-conversion crystal (PDC) by means of a dichroic mirror (MD1). This crystal length was chosen to provide a highly asymmetric \( f(\omega_1, \omega_2) \). After the crystal, the signal and idler polarizations are interchanged by two passes through a \( \lambda/4 \) plate located in front a mirror (M2). The pump, signal, and idler beams retrace their paths through the crystal, with their optical path lengths made equal by adjusting mirror M1, which is mounted on a piezoelectric transducer (PZT).

Because the signal and idler polarizations have been interchanged, the \( e \)-polarized light that is sent back into the down-converter carries the spectrum belonging to the original \( o \) polarization, and vice versa; more generally, the joint emission amplitude spectrum for these pairs is switched from \( f(\omega_o, \omega_e) \) to \( f(\omega_e, \omega_o) \). When the path lengths are balanced, and when the losses and dispersion for the pump and the down-conversion are equalized with a glass compensator plate (C), it becomes impossible, in principle, to discern whether the photons that ultimately emerge from the PDC were generated from the first or second pass of the pump pulse through the crystal. The resulting photon pairs therefore carry the superposition of the joint emission amplitude spectra for each process, as in Eq. (4). The phase \( \theta \) is varied by changing the position of M1 with the PZT.

The quantum state describing the photon pairs created by a single pump pulse is

\[
|\psi\rangle = |\text{vac}\rangle + \eta \int d\omega_o d\omega_e f(\omega_o, \omega_e) \times [|\omega_o\rangle_x |\omega_e\rangle_y + e^{i\theta} |\omega_o\rangle_y |\omega_e\rangle_x],
\]

where we use the labels \( x \) and \( y \) to denote the two orthogonally polarized output modes, but retain the \( e \)
and labels for the down-conversion frequencies. The probability that a given pump pulse will produce a photon pair is \(|\eta|^2\).

For a normalized Gaussian pump spectrum with bandwidth \(\sigma\), the quantum-mechanical expression for the coincidence rate is

\[
R_{AB}(\delta \tau, \theta) = R_{AB0} \left[ \frac{1}{2} + \cos(\theta) \frac{\sqrt{\pi}}{Z} \operatorname{erf}\left( \frac{Z}{4} \right) - \left[ \cos(\theta) \left( \frac{1}{2} - \frac{|\delta \tau|}{\tau_-} \right) \right] e^{-\left(\frac{Z}{\tau_-}\right)^2} \right.
\]

\[
+ \frac{\sqrt{\pi}}{Z} \operatorname{erf}\left( \frac{Z}{2} \left( \frac{1}{2} - \frac{|\delta \tau|}{\tau_-} \right) \right) \right] \operatorname{rect}\left( \delta \tau; \frac{\tau_- - \tau_+}{2}, \frac{\tau_- + \tau_+}{2} \right),
\]

where \(Z \equiv \sigma(\tau_o + \tau_e)/\sqrt{2}\). Here \(\tau_o\) is the maximum difference between the group delays experienced by the pump pulse and the \(o\) wave (after traveling the length of the PDC), \(\tau_e\) is the analogous quantity for the \(e\) wave, \(\tau_- \equiv \tau_e \tau_o\), and \(R_{AB0}\) is the mean coincidence rate for \(|\delta \tau| > \tau_-/2\).

The coincidence rate given by Eq. (6) is plotted in Fig. 2 for two fixed values of \(\theta\). By choosing appropriate M1 positions, the joint emission amplitude spectrum can be made symmetric (\(\theta = 0, \pm 2\pi, \ldots\)) or antisymmetric (\(\theta = \pm \pi, \pm 3\pi, \ldots\)) under the exchange in Eq. (3). As shown in Fig. 2, the former corresponds to the full restoration of the dip in coincidence counts as \(\delta \tau\) approaches zero, while the latter leads to a peak. In both cases, the existence of completely destructive (or constructive) quantum interference implies that the coincidence paths have been rendered fully indistinguishable by the symmetrization (or antisymmetrization) procedure.

For fixed values of \(\delta \tau\), Eq. (6) predicts sinusoidal oscillations with \(\theta\) in the coincidences as the source is changed from the symmetric to the antisymmetric configurations. The visibility of these fringes is determined by \(\delta \tau\), and can reach 100% for the peak-to-dip transitions at \(\delta \tau = 0\). We recorded these fringes for 11 different values of \(\delta \tau\), covering the range \(-0.16 \, \text{ps} < \delta \tau < 0.16 \, \text{ps}\). Counts were collected for 200 seconds at each \(\theta\) value and corrected for drifts in pump power.

The maxima and minima implied by the sinusoidal fits for all 11 values of \(\delta \tau\) are shown in Fig. 3. The lines are the result of a calculation for a 5 mm BBO crystal, scaled to the mean counting rate. The calculation includes the effects of a \(-10 \, \mu\text{m}\) imbalance in the interferometric path lengths (an artifact of our alignment procedure), as well as differences in the down-conversion intensity and dispersion for the two processes. Despite these systematic errors, the improvement in the fourth-order interference is evident here, as the coincidence rate in the center of the dip falls well below that of the unsymmetrized source (indicated by the square point on the dotted-dashed line). The maximum observed interference visibility in the center of the dip was 64%. This effect cannot be accounted for by interference...
modulation of the down-conversion rate, for although a small (2%–3%) oscillation with $\theta$ in the single-channel counts is indeed predicted and observed, it is out of phase with the coincidence modulation. Thus, the modulation of the coincidence rate is a genuine fourth-order interference effect that occurs because the double-detection paths are much less spectrally distinguishable than either single-channel detection path alone (as in many “quantum-eraser” experiments.

Finally, we note that this symmetrization technique may also be used to generate highly entangled states from sources which initially produce unentangled ones. In Eq. (1), the state $|\psi\rangle$ is unentangled if $f(x, y) = g(x)h(y)$, since it then factorizes into a product of the individual particle states. On the other hand, the state $|\psi\rangle_s$ in Eq. (2) will always be highly entangled, provided $g(x)h(y) \neq g(y)h(x)$, i.e., the amplitude function is not symmetric with respect to interchange of its arguments. In fact, the most entangled state occurs if either of these products is zero for some $\{x, y\}$ while the other is not, since then the combination $[g(x)h(y) + e^{i\theta}g(y)h(x)]$ is farthest from being a factorable function.

This method is general enough to allow the entanglement of both the space-time and polarization components of the state vector, and is especially useful for situations where the single-pass state is not highly entangled, as is often the case in parametric down-conversion. An intriguing feature of space-time entanglement is that there are potentially a large number of modes in which the photons may reside—many more than the two associated with the polarization modes normally used for quantum information processing—and this raises the question as to whether it is possible to make use of these extra degrees of freedom for such processing. For example, the spectral entanglement created here could be used in a frequency-domain analog of the time-domain cryptography scheme recently proposed by Brendel et al. [19].

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