Dispersion Penalty for 1.3-μm Lightwave Systems with Multimode Semiconductor Lasers

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Abstract—The effect of fiber dispersion on the performance of lightwave systems is analyzed for the case where multimode semiconductor lasers operating near the zero-dispersion wavelength of the single-mode fiber are used as sources. Both the intersymbol interference and the mode-partition noise are considered in the discussion of dispersion-induced power penalties. The theory is in good agreement with an experiment in which the bit error rate is measured for four lasers at various bit rates. The tolerable limits on the deviation of the laser wavelength from the zero-dispersion wavelength are obtained for a 1.3-μm system operating at 1.7 Gbit/s. Monte Carlo simulations are used to predict the effect of mode-partition noise on the performance of such high-speed lightwave communication systems.

I. INTRODUCTION

It is well known [1], [2] that the group-velocity dispersion in single-mode fibers limits the performance of optical communication systems. Since the group-velocity dispersion vanishes at a specific wavelength $\lambda_D$, referred to as the zero-dispersion wavelength (ZDWL), the dispersive effects can be minimized by operating the lightwave system in the ZDWL vicinity. However, as an exact match between the laser wavelength $\lambda_0$ and the ZDWL $\lambda_D$ is practically not feasible, the system performance depends on the wavelength deviation $\Delta \lambda = |\lambda_0 - \lambda_D|$. Since the dispersion penalty would generally vary from one fiber link to another depending on the values of the fiber and laser parameters, the goal of the system designer is to set the tolerance limits on both $\lambda_0$ and $\lambda_D$ so that the distribution of dispersion penalties is below some predetermined level. The tolerance limits, however, depend on other laser and system parameters such as the bit rate $B$, the fiber length $L$, and the spectral width $\sigma$ of the multimode semiconductor laser.

Since an unnecessarily tight specification on the laser wavelength is detrimental to laser yield, it is prudent to choose the wavelength window $\lambda_0 \pm \Delta \lambda$ as wide as possible within the system requirements. One objective of this paper is to consider the dependence of the dispersion-induced power penalty [3]–[6] on various parameters such as $B$, $L$, $\sigma$, and $\Delta \lambda$, and determine the tolerable $\Delta \lambda$ for a specific fiber link. The other objective is to determine the distribution of power penalties for the whole system by using Monte Carlo simulations by including the allowed link-to-link variations in the system parameters. Such simulations can then be used to predict the median and worst-case penalties. The results are applied to a specific 1.3-μm lightwave system operating at 1.7 Gbit/s to determine the tolerable wavelength deviation $\Delta \lambda$.

II. DISPERSION-INDUCED POWER PENALTY

There are two basic mechanisms through which the dispersion can induce power penalties for transmission systems making use of single-mode fibers.

1) The broadening of an optical pulse leads to intersymbol interference. Qualitatively speaking, the power loss occurring within the bit period $B^{-1}$ has to be compensated by increasing the launched power in order to maintain the required signal-to-noise ratio (SNR) at the receiver.

2) Mode-partition noise in multimode semiconductor lasers (due to power redistribution among laser modes with the total power relatively constant) coupled with the fiber dispersion reduces the SNR at the receiver. This mechanism under some conditions can degrade the SNR so much that the system cannot achieve a specified bit error rate even by increasing the launched power (i.e., a bit-error-rate floor is observed).

A. Intersymbol Interference

It is generally difficult to estimate the amount of power penalty arising from the pulse broadening since it depends on details of the laser transmitter and the receiver. To develop a qualitative understanding, assume that the laser transmits a Gaussian pulse whose spectrum is also Gaussian, i.e.,

$$P(\lambda, t) = \frac{P_0}{\sigma \sqrt{2\pi}} \exp\left[-\frac{(\lambda - \lambda_0)^2}{2\sigma^2}\right] \exp\left[-\frac{t^2}{2T_0^2}\right]$$

(1)

where $\sigma$ is the spectral half-width, $T_0$ is the temporal half-width, and $P_0$ is the peak power. It can be shown [7] that the pulse remains Gaussian after propagation inside the fiber and the transmitted power (integrated over all wavelengths) at the receiver is given by

$$P_{tr}(t) = P_0 \left(\frac{T_0}{T}\right) \exp\left[-\frac{t^2}{2T^2}\right]$$

(2)
where
\[ T = T_0 \left[ 1 + \left( DL \sigma / T_0 \right)^2 \right]^{1/2}. \] (3)

\( D \) is the dispersion (in ps/km-nm), and \( L \) is the fiber length. To relate \( T_0 \) to the bit rate \( B \), we use the relation \( T_0 / 2\pi = B^{-1} \) obtained by equating the Gaussian-pulse area to that of a square pulse of duration \( B^{-1} \). The broadening factor of the pulse is then given by
\[ T / T_0 = \left[ 1 + 2\pi (BDL \sigma)^2 \right]^{1/2}. \] (4)

A simple estimate of the dispersion penalty \( \alpha_{\text{SSI}} \) due to intersymbol interference can be obtained by comparing the received power with and without dispersion at the decision point (assumed to be located at the pulse center). We then obtain (in decibels)
\[ \alpha_{\text{SSI}} = 10 \log \left( T / T_0 \right) = 5 \log \left[ 1 + 2\pi (BDL \sigma)^2 \right]. \] (5)

Since in all cases of practical interest \( BDL \sigma \ll 1 \), (5) can be approximated by
\[ \alpha_{\text{SSI}} \approx 14 (BDL \sigma)^2. \] (6)

The preceding estimate of \( \alpha_{\text{SSI}} \) is based on an oversimplified model. A more accurate calculation of the intersymbol-interference effects should take into account receiver details, in particular those related to the equalizer, and requires numerical simulations \([8, 9]\). In the following discussion, we use (5) to estimate the power penalty \( \alpha_{\text{SSI}} \). As a rough criterion, note from (6) that
\[ |BDL \sigma| < 1/4 \] (7)
in order to keep \( \alpha_{\text{SSI}} \) at a level near or below 1 dB.

B. Mode-Partition Noise

Ogawa has studied in detail the power penalty due to mode-partition noise (MPN) using a simple model \([3, 5]\). We briefly discuss this model. It is assumed that the total power carried by each pulse is constant while the power in each mode fluctuates, i.e.,
\[ \sum_{i=1}^{N} A_i = 1 \] (8)

where \( A_i(t) \) is a random variable denoting the instantaneous power in the \( i \)th longitudinal mode. \( N \) is the total number of longitudinal modes, and the total power has been normalized to unity. The measured time-averaged power spectrum \( p(\lambda_i) \) shows the average value of the mode power at the mode-wavelength \( \lambda_i \):
\[ \bar{A}_i = p(\lambda_i) = \int \ldots \int A_i(p(A_1, A_2, \ldots, A_N) \cdot dA_1 dA_2 \ldots dA_N \] (9)

where \( p(A_1, A_2, \ldots, A_N) \) is the joint probability distribution function.

When the optical pulse is transmitted through the dispersive fiber, each mode is delayed by a different amount because of group-velocity dispersion. If \( f(\lambda_i, t) \) denotes the received waveform for \( i \)th mode, the total received signal is
\[ r(t) = \sum_{i=1}^{N} f(\lambda_i, t) A_i. \] (10)

The MPN leads to an additional noise at the receiver with the variance (evaluated at the decision time slot \( t = t_0 \)).
\[ \sigma_{\text{MPN}}^2 = r^2(t_0) - \left[ r(0) \right]^2. \] (11)

This noise is added to the receiver noise so that the effective SNR \( Q \) is determined by
\[ \frac{1}{Q^2} = \left( \frac{\sigma_r}{S} \right)^2 + \left( \frac{\sigma_{\text{MPN}}}{1} \right)^2. \] (12)

where \( \sigma_r \) is the total receiver noise and \( S \) is the received signal.

The MPN-induced power penalty is related to the increase in the received power that is necessary to maintain a constant SNR. Since in the absence of MPN, \( S_0 = Q r \), is the required power, the power penalty (in decibels) is given by
\[ \alpha_{\text{MPN}} = 10 \log \left( \frac{S}{S_0} \right) = 5 \log \left( \frac{1}{1 - Q^2 \sigma_{\text{MPN}}^2} \right). \] (13)

The value of \( Q \) is determined by the maximum acceptable bit error rate (BER) using \([6]\)
\[ \text{BER} = \frac{1}{Q \sqrt{2\pi}} \exp \left( -Q^2/2 \right). \] (14)

\( Q \approx 6.7 \) if \( 10^{-11} \) is chosen as the maximum bit error rate. Equation (13) shows that the power penalty becomes infinite for
\[ \sigma_{\text{MPN}} \geq Q^{-1} \approx 0.15 \]

implying that a bit error rate of \( 10^{-11} \) cannot be achieved in that case (an error-rate floor above \( 10^{-9} \)). It is this feature of MPN that makes it of critical importance in the design of lightweight transmission systems.

C. Evaluation of \( \sigma_{\text{MPN}} \)

The evaluation of \( \sigma_{\text{MPN}} \) using (11) requires the knowledge of the joint probability distribution function \( p(A_1, A_2, \ldots, A_N) \) which is generally unknown. Ogawa has introduced the concept of the mode partition coefficient \( k \) defined by \([6]\)
\[ k^2 = 1 - \alpha = \frac{\bar{A}_i^2 - \bar{A}_i \bar{A}_j}{\bar{A}_i \bar{A}_j} \] (for any \( i \) \( \neq j \)).

where it is assumed that \( \bar{A}_i \bar{A}_j = \alpha \bar{A}_i \bar{A}_j \) for all modes \( (i \neq j) \). The assumption that all modes have the same cross-correlation with respect to each other is the most critical assumption of this model and may not always hold for semiconductor lasers. Its use however allows the evaluation of \( \sigma_{\text{MPN}} \) without the knowledge of \( p(A_1, A_2, \ldots, A_N) \).
Using (10), (11), and (15), one finds that
\[ \sigma_{\text{mpn}} = k \left[ \sum_{i=1}^{N} f_i^2 \hat{A}_i - \left( \sum_{i=1}^{N} f_i \hat{A}_i \right)^2 \right]^{1/2} \]  
(16)
where \( f_i = f(\lambda_i, t_0) \). We assume that the received signal waveform after equalization at the receiver is of the form
\[ f_i = f(\lambda_i, t_0) = \cos \left[ \pi B (t_0 + \Delta \tau_i) \right] \]  
(17)
where
\[ \Delta \tau_i = LD(\lambda_i - \lambda_0) \]  
(18)
is the relative delay of the \( i \)th mode with respect to the central mode at \( \lambda_0 \) during propagation for a fiber of length \( L \) and dispersion \( D \). The decision circuit samples the received signal at times \( t_0 = N/B \), where \( N \) is an integer, and therefore \( f_i = \pm \cos (\pi B \Delta \tau_i) \) in (17).

The numerical value of \( \sigma_{\text{mpn}} \) depends on the steady-state spectrum \( \hat{A}_i \) which we assume to be Gaussian:
\[ \hat{A}_i = \rho(\lambda_i) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(\lambda_i - \lambda_0)^2}{2\sigma^2} \right]. \]  
(19)
The calculation is considerably simplified if we assume that the discrete sum in (16) can be replaced by an integral. The final result is
\[ \sigma_{\text{mpn}} = \frac{k}{\sqrt{2}} \left[ 1 - \exp \left( -\beta^2 \right) \right] \]  
(20)
where
\[ \beta = \pi B D L \sigma \]  
(21)
is a dimensionless parameter. Our result (20) is a slight generalization of the Ogawa’s result [3], [6] since we did not assume \( \pi B \Delta \tau_i \ll 1 \) in (17). In the limit \( \beta \ll 1 \)
\[ \sigma_{\text{mpn}} = k \beta^2 / \sqrt{2} \]  
(22)
and we recover the Ogawa’s result. The calculation of \( \sigma_{\text{mpn}} \) can be generalized to include the effect of non-Gaussian spectrum by using the discrete sum in (16). We calculate our results for the MPN-induced power penalty using (13) and (20).

The expression (20) for the MPN noise shows that \( \sigma_{\text{mpn}} \) is directly proportional to the mode-partition coefficient \( k \) defined by (15). The numerical value of \( k \) is relatively uncertain and may depend on a large number of parameters such as the bit rate (through the pulse duration), the modulation depth (through the bias level), and the spectral width \( \sigma \). Experimental measurements suggest a value of \( k = 0.5 \), and this value has been widely used [6]. It should however be stressed that the experimental data have been obtained for relatively wide pulses (\( > 1 \) ns) and depend to some extent on the method of measurement.

III. RESULTS FOR 1.7-GBIT SYSTEM

In this section we present the results for a 1.3-\( \mu \)m lightwave transmission system operating at a bit rate of 1.7 Gbit/s. The dispersion parameter \( D \) is obtained assuming that in the ZDWL vicinity \( D \) varies linearly with the wavelength deviation, i.e.,
\[ D = S(\lambda_0 - \lambda_D) \]  
(23)
where the slope \( S = \partial D / \partial \lambda \) is evaluated at the ZDWL \( \lambda_D \). The numerical values of the average slope and its standard deviation are obtained from measurements [10] on a large number of fiber segments (sample size of 6538) varying in length from 2-7 km. Table I shows the average values and the range of variations in the values of the parameters which control the dispersion penalty. In particular, the fiber length \( L \) is assumed to be uniformly distributed over the range 36-46 km. The other parameters \( S, \lambda_D, \lambda_0, \) and \( \sigma \) are assumed to have a Gaussian distribution whose standard deviation is shown in Table I (under the range column). These parameter values are typical of realistic lightwave systems. To quantify the system performance, we consider both the median and the worst-case penalties. The median penalty is calculated by using the average values in Table I. The worst-case penalty is calculated by taking \( L = 46 \) km (the maximum fiber length), \( S = 0.102 \) ps/km/nm, and \( \sigma = 2.5 \) nm. The values of \( S \) and \( \sigma \) correspond to 2 standard deviations above the average value of the corresponding Gaussian distribution. For this choice the probability of a larger penalty than that calculated here is \( < 10^{-3} \).

Consider first the dispersion penalty \( \alpha_{\text{disp}} \) due to intersymbol interference. Using the average parameter values from Table I in (5), we find that \( \alpha_{\text{disp}} \leq 0.5 \) dB as long as \( |\lambda_0 - \lambda_D| < 16 \) nm. If we use (5) for the worst-case values of \( L = 46 \) km, \( S = 0.102 \) ps/km/nm², and \( \sigma = 2.5 \) nm, we find that for \( \alpha_{\text{disp}} \leq 0.5 \) dB the wavelength window narrows down to
\[ |\lambda_0 - \lambda_0| < 10 \text{ nm} \]  
(24)
This result shows that the intersymbol interference does not cause significant power penalty for wavelength deviations as large as 10 nm.

We next consider the dispersion penalty due to MPN. The average power penalty \( \alpha_{\text{mpn}} \) for a bit error rate of \( 10^{-11} \) as a function of \( \lambda_0 \) is shown in Fig. 1 using the average parameter values given in Table I and (13), (20), and (21). Since the value of the mode-partition coefficient \( k \) is relatively uncertain, curves for \( k = 0.5 \) and \( k = 1 \) are shown. The power penalty increases rapidly with an increase in the wavelength deviation \( |\lambda_0 - \lambda_D| \) and eventually becomes infinite (an error-rate floor at a level higher than a \( 10^{-11} \) bit error rate).

The wavelength window \( |\lambda_0 - \lambda_0| \) that \( \alpha_{\text{mpn}} \) achieves be a preselected level can be obtained as follows. If we use (22) and (23), we obtain
\[ |\lambda_0 - \lambda_D| = \left( \frac{\sqrt{2} \sigma_{\text{mpn}}}{k} \right)^{1/2} \frac{1}{\pi B D L \sigma} \]  
(25)
where from (13) \( \sigma_{\text{mpn}} \) is related to \( \alpha_{\text{mpn}} \) by
\[ \sigma_{\text{mpn}} = \frac{1}{6.7} \left( 1 - 10^{-\alpha_{\text{mpn}}/5} \right)^{1/2} \]  
(26)
and we took $Q = 6.7$ corresponding to a bit error rate of $10^{-11}$.

To provide the worst-case estimates of $\alpha_{\text{mpn}}$ for comparison with the median $\alpha_{\text{mpn}}$, Fig. 2 shows $|\lambda_0 - \lambda_D|$ as a function of $\alpha_{\text{mpn}}$ using $L = 46$ km, $S = 0.102$ ps/km/nm$^2$, and $\sigma = 2.5$ nm. We note that in order to keep the dispersion penalty (for $k = 1$) below 0.5 dB:

$$|\lambda_0 - \lambda_D| < 5 \text{ nm.}$$

We can also calculate the wavelength window for the average penalty to be below 0.5 dB and find that (Fig. 1, $k = 1$)

$$|\lambda_0 - \lambda_D| < 8 \text{ nm.}$$

These ranges of wavelength deviation should be compared with (24). It is evident that the MPN is the limiting factor rather than the intersymbol interference and sets the final limit on the wavelength deviation $|\lambda_0 - \lambda_D|$ that can be tolerated, given these parameters.

IV. EXPERIMENTAL RESULTS

Since the dispersion penalties in Figs. 1 and 2 depend critically on the relatively unknown value of the model parameter $k$, system experiments were done using four lasers of varying wavelengths. As the dispersive effects depend on the product $BDL\sigma$ [see (20) and (21)], the power penalty curves can be obtained by varying any one of the four parameters. We have found it convenient to vary the bit rate $B$. Fig. 3 shows the measured power penalty for a bit error rate of $10^{-10}$ as a function of the bit rate for the four lasers. Calculated curves show the theoretical fit to the experimental data. The laser wavelength $\lambda_D$ and the spectral width $\sigma$ were obtained using the longitudinal-mode spectrum for each laser measured under the operating conditions. The measured fiber parameters were $L = 28.1$ km, $\lambda_D = 1306$ nm, and $S = 0.09276$ ps/km/nm$^2$. To obtain the fit shown in Fig. 3, it was necessary (1) to set $k = 1$ and (2) to choose the ZDWL 4.5 nm shorter than the measured value. The fitted value of $\lambda_D$ is within the experimental uncertainty. The power penalty $\alpha_{\text{mpn}}$ predicted using $k = 0.5$ is significantly smaller than the measured values, and therefore $k = 1$ was used in the Monte Carlo simulations discussed in the next section.

It should be stressed that actual lasers are not expected to have $k = 1$ since it implies that at any time the laser output is concentrated in one mode only. The experimental measurements shown in Fig. 3 indicate that (20) underestimates the extent of MPN. The reason behind this underestimation can be traced back to the Gaussian form (19) for the mode spectrum. Although the Gaussian-spectrum assumption is often used and provides a reasonable fit for the central part of the experimental spectra, its use ignores the contribution of the spectral modes in the wings. If we use the discrete sum (16) in place of (20) and include the modes up to 30 dB down from the dominant mode, the experimental results can be fitted with a lower value of the mode-portion coefficient $k = 0.7$. Our recent work shows that the experimental spectra can be fitted by assuming a super-Lorentzian profile [11]. The use of this form has also resulted in $k \approx 0.7$. In our Monte Carlo simulations we have nonetheless used $k = 1$ in order to provide the worst-case estimates of the power penalty that are independent of the actual value of $k$.
found to be negligible (≈0.02 dB). However, the variation is quite large, with each change of one standard deviation producing an order of magnitude change in penalty. Still, the power penalty is below 0.5 dB in 92 percent of the cases. The transmitters for which the dispersion penalty exceeds the system-allocated margin can be identified through the BER measurements using a fiber with nearly worst-case parameters. Our simulations results show that a wavelength window of 5 nm centered at the ZDWL (λ₀ = λD ± 5 nm) can be used for 1.7 Gbit/s with only a few percent yield drop in transmitters.

VI. CONCLUSIONS

We have analyzed the effect of intersymbol interference and mode-partition noise on the performance of high-speed lightwave systems operating near the zero-dispersion wavelength of single-mode fibers. It is found that the mode-partition noise is the limiting mechanism and sets the ultimate limit on the tolerable range of laser wavelengths. For a 1.3-μm system operating at 1.7 Gbit/s, the acceptable range is λD ± 5 nm. The theory was compared with the experiments by measuring the bit-error rates for four lasers as a function of the bit rate. The Gaussian-spectrum assumption is found to underestimate the power penalty. We performed Monte Carlo simulations to study the overall system performance. Although we considered only 1.3-μm systems, the analysis can be readily applied to the case of 1.55-μm systems making use of dispersion-shifted fibers and multimode semiconductor lasers.

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REFERENCES

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