Asymmetric Channel Gain and Crosstalk in Traveling Wave Optical Amplifiers

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Abstract—Previous studies have shown that in multichannel optical amplification using a traveling wave amplifier, nonlinear frequency mixing effects can arise due to the modulation of the carrier density. It was shown that in the small-signal case, the carrier density modulation leads to a weak asymmetry in the gains experienced by individual channels. We extend the theoretical results here by including the effects of gain saturation. We find, in the two-channel case, that the gain asymmetry can be as large as 10 dB depending on the channel separation, carrier lifetime, and input powers. We also discuss the implications of this on the choice of modulation scheme.

I. INTRODUCTION

It is commonly assumed in multichannel optical amplification that all channels experience the same gain when they travel through an ideal traveling wave amplifier (TWA) which has a constant gain profile (as opposed to a Fabry–Perot amplifier with periodic gain peaks). It has been shown [1], [2] that this is true only when channels are spaced far apart. When the channel separation is comparable to the inverse of the carrier lifetime, it was shown [2] that nonlinear effects can occur which cause individual channels to experience different degrees of gain and also lead to the generation of unwanted frequency components. The physical mechanism giving rise to this nonlinear effect is the modulation of the carrier density at the beat frequency, in the presence of a number of channels. This carrier density modulation leads to nonlinear (induced dipole) polarizations and when these are used as source terms in the nonlinear wave equation, we find that the growth of the waves is modified resulting in gain and phase changes.

However, previous work concentrated on the small-signal regime in which gain saturation is neglected. These results showed that for a two-channel system operating in the small-signal regime, a weak asymmetry in the gain exists which favors the channel with the smaller frequency. In this paper, we study the case when practical values of output power are obtained from the amplifier, and thus, we are working in a regime in which the effects of gain saturation have to be considered along with the nonlinear carrier density modulation. We find that the gain asymmetry remains when gain saturation is present and that the difference in gain may be pronounced (for example, 10 dB) depending on the channel separation, carrier lifetime, and input powers. We also find that the variation in gain gives rise to crosstalk which is more severe when ASK is used rather than a constant envelope modulation. We are currently investigating the more complicated case when an arbitrary number of channels (\( > 2 \)) are present and we will report the results in a future paper.

II. THEORETICAL FRAMEWORK

We will summarize here the physical mechanism (as discussed in [1]) that gives rise to the nonlinear effects discussed in Section I. Let us assume that two waves at frequencies \( \omega_1 \) and \( \omega_2 \) (\( \omega_1 < \omega_2 \)) and with complex amplitudes \( E_1 \) and \( E_2 \) are propagating inside the amplifier. These waves are given by

\[
E_1 = \hat{x}[E_1 e^{-i(\omega_1 t - k_1 \cdot r)} + \text{c.c.}]
\]

and

\[
E_2 = \hat{x}[E_2 e^{-i(\omega_2 t - k_2 \cdot r)} + \text{c.c.}]
\]

where \( \hat{x} \) is the polarization unit vector, \( \omega_1 \) and \( \omega_2 \) are the angular frequencies, \( k_1 \) and \( k_2 \) are the wave vectors of the two waves, and c.c. denotes the complex conjugate. The superposition of these two waves gives rise to a space-time grating of the intensity

\[
|E|^2 = |E_1|^2 + |E_2|^2 + |E_1^* E_2 e^{-i(\Omega t - (k_2 - k_1) \cdot r)} + \text{c.c.}|
\]

where \( \Omega = \omega_2 - \omega_1 \) is the angular frequency difference. The term in brackets corresponds to a spatial dynamic grating which produces a corresponding grating in the parameters of the medium if they depend on the intensity. In this case, the field intensity influences the electron density and a dynamic grating of the electron density is established, which in turn causes changes in the refractive index and gain.

The carrier density of electrons \( N \) in the active region can be found by solving the rate equation [3], [4]:

\[
\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_e} - \frac{g(N)}{\hbar \omega} \langle |E|^2 \rangle
\]

where \( I \) is the current flowing through the active region of volume \( V \), \( q \) is the electron charge, \( \tau_e \) is the carrier lifetime, and the angle brackets denote averaging over the
transverse coordinates. The axial variation of $N$ along the propagation direction is retained in view of the growth of channel intensities. In (4) we have neglected the diffusion process, which is justified if the diffusion coefficient $D$ satisfies the condition $D \leq \Omega / |k_1 - k_2|^2$. This condition is easily satisfied when $k_1$ and $k_2$ are close to one another (that is, when the interacting waves are propagating along the same or similar directions). The gain $g(N)$ is assumed to vary linearly with the carrier density [5], that is:

$$g(N) = \Gamma a (N - N_0)$$

(5)

where $\Gamma$ is the confinement factor, $a (= 2 \times 3 \times 10^{-16}$ cm$^2$) is the gain coefficient and $N_0 (= 1 - 2 \times 10^{18}$ cm$^{-3}$) is the carrier density at which the active region becomes transparent (onset of population inversion).

Substituting the field intensity given by (3) in the carrier rate equation (4), we obtain an approximate solution in which the carrier density $N$ has a constant component $\bar{N}$ and an oscillatory component of magnitude $\Delta N$, that is:

$$N = \bar{N} + (\Delta N e^{-i\omega t} + \text{c.c.}).$$

(6)

The oscillation in carrier density gives rise to oscillations of the optical characteristics of the medium, such as gain and refractive index. The values for $\bar{N}$ and $\Delta N$ can be easily found by solving for the steady state in (4), and are given by

$$\bar{N} = I/I_0 + P_1/P_2 + P_2 N_0 / (1 + P_1 + P_2)$$

(7)

and

$$\Delta N = -\bar{N} (\frac{\Delta N}{N_0}) E_j^* E_j e^{-i(k_1 - k_2)z}/P_S$$

(8)

where $I_0 = qVN_0/\tau_c$ is the current needed to achieve transparency, $P_S = qN_0/\tau_c$, is the saturation power,$^4$ and $P_j = |E_j|^2/P_S$ is the normalized power in the $j$th channel ($j = 1, 2$). The saturation power $P_S$ is the small-signal saturation power, that is, the power at which the small-signal gain coefficient drops to half its unsaturated value. This is not to be confused with the saturation power defined at the output, which is the power at which the total gain drops by half. Typically $P_S \approx 5$-10 mW.

When the semiconductor medium interacts with the optical field, a macroscopic polarization is induced in the medium. In general, we must consider the dynamics of the induced polarization. However, for a semiconductor laser amplifier the polarization lifetime $T_2$ governed by the intraband scattering, is much shorter than other time scales of interest such as the photon and carrier lifetimes. In the so-called rate-equation approximation, the medium is assumed to respond instantaneously to the optical field, and the induced polarization is given by

$$\vec{P} = \epsilon_0 \chi(N) \vec{E}$$

(9)

where $\chi$ is the susceptibility, $\vec{E}$ is the total electric field given by

$$\vec{E} = \hat{\chi} \left[ \sum_{j=1}^{2} E_j e^{-i(\omega t - k_2 z)} + \text{c.c.} \right]$$

(10)

and $\hat{\chi}$ is the polarization induced by the field and given by

$$\hat{\chi} = \hat{\chi} \left[ \sum_{j=1}^{2} \vec{P}_j e^{-i(\omega t - k_2 z)} + \text{c.c.} \right].$$

(11)

In (10) and (11) we have neglected the four-wave-mixing components created at frequencies $\omega_1 - \Omega$ and $\omega_2 + \Omega$ on each side of the amplified channels. More detailed calculations show that their effect on the growth of the components at $\omega_1$ and $\omega_2$ is small. Now, to complete our description of the nonlinear optical interaction we need to specify the relation between the polarization components and the optical fields. In our case, the susceptibility has two parts, the background unpumped material susceptibility and another part which depends on the population difference and is given by [6]:

$$\chi(N) = \frac{-\hbar\epsilon_0}{\omega} (\alpha + i) g(N)$$

(12)

where $\eta$ is the refractive index, and $\alpha$ is the linewidth enhancement factor (the ratio of the change of the real part to the change of the imaginary part of the refractive index with carrier density). Using (5) and (6) we see that the gain $g(N)$ is given by

$$g(N) = \Gamma a (\bar{N} - N_0) + \Gamma a (\Delta N e^{-i\omega t} + \text{c.c.}).$$

(13)

When (10)-(13) are substituted in (9) and terms at $\omega_1$ and $\omega_2$ are collected, we get expressions that relate the polarizations at the two frequencies with the complex field amplitudes. These expressions are given by

$$\vec{P}_1 = \frac{-\epsilon_0 \chi_0}{\omega} \left[ \sum_{j=1}^{2} \frac{1}{1 + P_1 + P_2} \frac{E_j}{N_0 (1/I_0 - 1 + P_1 + P_2)} + \text{c.c.} \right]$$

(14)

and

$$\vec{P}_2 = \frac{-\epsilon_0 \chi_0}{\omega} \left[ \sum_{j=1}^{2} \frac{1}{1 + P_1 + P_2} \frac{E_j}{N_0 (1/I_0 - 1 + P_1 + P_2)} + \text{c.c.} \right]$$

(15)

where $\chi_0 = \Gamma a N_0 (1/I_0 - 1)$ is the small-signal gain, and $\Omega$ is the normalized channel spacing in radians (that is, the actual channel spacing normalized by the carrier lifetime).
We use the polarizations as driving terms in the nonlinear wave equation
\[
\frac{\partial^2 E(z, t)}{\partial z^2} - \frac{n^2 \partial^2 E}{c^2 \partial t^2} = \frac{1}{\alpha_0} \frac{\partial^2 P}{\partial t^2}
\]  
(16)
in order to find a differential equation for the field at each frequency and from which we can get the gain coefficient [7]. When this is done, using the slowly varying envelope approximation we find that the gain coefficient at $\omega_i (i = 1, 2)$ is simply given by the imaginary part of the susceptibility that relates $\tilde{P}_i$ and $\tilde{E}_i$. The real part of the susceptibility designates the refractive index. The gain coefficients for the two channels are then given by [2]:
\[
g_1 = \frac{g_0}{1 + P_1 + P_2} \left[ 1 - \frac{(1 + P_1 + P_2 - \alpha \Omega \tau) P_2}{(1 + P_1 + P_2)^2 + (\Omega \tau)^2} \right]
\]
(17)
\[
g_2 = \frac{g_0}{1 + P_1 + P_2} \left[ 1 - \frac{(1 + P_1 + P_2 + \alpha \Omega \tau) P_1}{(1 + P_1 + P_2)^2 + (\Omega \tau)^2} \right]
\]
(18)
For an ideal TWA we can neglect the effect of facet reflections and then, in order to find the amplifier gain, we need to solve the two coupled differential equations
\[
\frac{dP_n}{dz} = g_1 P_2
\]
(19)
and
\[
\frac{dP_2}{dz} = g_2 P_2
\]
(20)
with the boundary condition $P_i(0) = P_i^{IN} (i = 1, 2)$. In (19) and (20), the waveguide loss is assumed to be negligible compared with $g_1$ and $g_2$. The solution gives the output power $P_n^{OUT}$, and the single-pass amplifier gain is obtained using $G_i = P_n^{OUT}/P_i^{IN}$.

In [2], equations (19) and (20) were solved analytically for the small-signal case in which $1 + P_1 + P_2$ was replaced by 1. In this paper, we obtain a numerical solution in order to study the nonlinear effects at high output signal levels.

III. Results

All the results in this section are based on the numerical solution of (19) and (20) with the gains $g_1$ and $g_2$ given by (17) and (18). We assume a small-signal single-pass gain $G_0 = \exp (g_0 L)$ of 25 dB and a linewidth enhancement factor $\alpha = 6$. The qualitative behavior of the results will not change if different values for $G_0$ and $\alpha$ are chosen.

We can see from the signs of the $\alpha \Omega \tau$ terms in (17) and (18) that the carrier density modulation effect causes an increase in the gain at $\omega_1$ due to the presence of the tone at $\omega_2$ and correspondingly a reduction in the gain seen at $\omega_2$ due to the presence of the tone at $\omega_1$. This effect has been experimentally verified recently [8]. The carrier lifetime was found to be of the order of 0.2 ns which is about an order of magnitude smaller than the typical carrier lifetime in semiconductor lasers. Other studies on intermodulation distortion in optical amplifiers [9] have yielded results consistent with a carrier lifetime of 0.28 ns. The reduction in carrier lifetime is due partly to an increase in Auger recombinations because of a relatively high carrier density in high-gain amplifiers.

The individual channel gains when the power in one channel is increased while that in the other channel is fixed are shown in Figs. 1 and 2. In Fig. 1, $P_n^{IN}$ is fixed while in Fig. 2 the scenario is reversed. In each figure, the left and right columns show results for different channel separations taking $\Omega \tau$ as 0.2 and 1. In each column, the effect of input power on the channel gains is shown by considering three values of $P_n^{IN}$. We can see from Figs. 1 and 2 that $G_2$ is always smaller than $G_1$. This is due to the carrier density modulation effect, in the absence of which, both channels will experience gain saturation and no gain asymmetry will arise [2]. The situation is analogous to stimulated scattering off an index grating, where one channel acts as a pump for the other and gain asymmetry arises depending on whether the pump is in the Stokes or anti-Stokes region. We observe from Fig. 1 that when the channels are close together, increasing the input power $P_2^{IN}$ in the high frequency channel can induce an additional gain at the lower frequency (the peak in Fig. 1) while when the channels are taken farther apart, this does not happen, and both channels will saturate as $P_2^{IN}$ is increased. We also observe that the gain imbalance can be as large as 10 dB. The effect just described and seen in Fig. 1 at a normalized channel spacing of 0.2 does not appear in Fig. 2 at the same separation. This is because $\omega_2$ is on the wrong side (red side) of $\omega_1$ to cause any additional gain at $\omega_2$ when the power at $\omega_1$ is increased.

The effects described above indicate that in a local-area network, situations in which users come in with a high power can be very detrimental to other users. Ideally, we would like all channels to have equal powers when they are amplified, but note that the issue of having equal powers or not is intimately related to the network topology, and some care must be exercised in deciding where to use an amplifier in a frequency division multiplexed network.

In practical situations, we are sometimes interested in the individual channel gains when a certain useful output power is extracted from the amplifier. Fig. 3 shows the individual channel gains versus total output power for equal input powers ($P_1^{IN} = P_2^{IN}$) and a range of channel separations. We can see that if we require, say, a normalized total output power of 0 dB (normalized by $P_2$), then we can have up to 9-dB variation in the channel gains for $\Omega \tau = 0.2 - 5$ (corresponding to a frequency separation $\approx 0.16 - 4$ GHz when $\tau_2 = 0.2$ ns). If we wish to reduce the gain imbalance that the two channels experience we have to back off and extract a smaller amount of power from the amplifier. If we use the 1-dB gain compression criterion (that is, the gain of either channel
does not change by more than 1 dB as the other channel is switched on or off) we get the curve shown in Fig. 4 which shows the total output power versus channel separation. We see that to meet this criterion we have to back off considerably and obtain a much smaller output power from the amplifier as channels get closer.

Fig. 5 shows the individual channel gains $G_1$ and $G_2$ as a function of the input power per channel for normalized
channel separations $\Omega_r = 0.2-5$. Also shown are the curves obtained when only gain saturation is considered for the single channel and the 2-channel case. We see, as channels are spaced farther apart, that the gain asymmetry decreases and eventually as $\Omega_r >> 1$ both channels will experience the same gain given by the 2-channel gain saturation curve.

Fig. 6 shows the individual channel gains $G_1$ and $G_2$ versus the normalized channel spacing $\Omega_r$, for several values of equal input powers. We also show $G_i$ and $G_j$ ob-
Fig. 3. (a)-(d) Gain versus total output power. This figure shows the individual channel gains versus total output power for equal input powers \(P_i^1 = P_i^2\) and a range of channel separations \(\Omega r_x\). The figure also shows the performance obtained when only gain saturation is considered.

Fig. 4. Total output power versus channel separation for 1-dB gain variation. The total output power versus frequency separation is shown when the 1-dB gain compression criterion is used (that is, the gain of either channel does not change by more than 1 dB as the other channel is switched on or off). To meet this criterion, we have to back off considerably and obtain a much smaller output power from the amplifier as channels get closer.

Tained with gain saturation only; these gains are independent of channel spacing and appear as horizontal lines in this figure. From Fig. 6 we can make some general observations about the effect of this gain asymmetry on system performance under different modulation schemes. For illustration purposes, consider Fig. 6(d) which has a high input power for a 25-dB amplifier. With amplitude-shift keying (ASK), either channel will be switched on and off to transmit data. Let us assume that the channel separation \(\Omega r_x = 3\). When both channels are on, they will experience different gains as shown by the black squares. If we consider channel 1 then as channel 2 is switched on and off, the gain \(G_1\) of the user at \(\omega_1\) switches between the black square and the empty square on the single-channel gain-saturation curve. Similarly, as channel 1 is turned on and off, the gain \(G_2\) of the user at \(\omega_2\) is modulated. The strength of this gain modulation (crosstalk) is worse for the high frequency user (\(\omega_2\)). As the channel separation is increased, the gains of both channels will tend asymptotically to the 2-channel gain saturation value. Now as
one channel is modulated, the other channel's gain switches between the two gain saturation levels for one and two channels, respectively. So we see, for ASK, we will have a residual crosstalk even if channels are spaced very far apart. This has been observed experimentally [10].

In ASK, the variation in gain can be translated easily to variation in signal level at the output of the detector and consequently this will lead to "eye closure." The decision threshold will also have to be set differently for each user as they experience different degrees of crosstalk, and thus, different bit error rates. The "eye closure" can be translated in a straightforward way into a bit error rate degradation.

On the other hand, let us now consider wide deviation frequency-shift keying (FSK) in which each user switches between two tones to transmit the data. The use of a large modulation index allows us to regard the transmitted spectrum as a two-tone spectrum. As each user modulates his frequency, the effective user separation is modulated. If the channel spacing is $\Omega_s$ and the individual tone separation is $\Omega_T$, then, for a given user, the other user's spacing can take the values $\Omega_s - \Omega_T$, $\Omega_s$, or $\Omega_s + \Omega_T$. Since the other user is always on but with a variable separation, the gain of the user at $\omega_1$ changes between extremes shown by triangles on $\omega_1$'s gain curve in Fig. 6 and similarly for the user at $\omega_2$. We see that the user at $\omega_2$ has consistently a smaller gain and that the gain variation for either user is much smaller than the case of ASK. As the channel separation increases, the degree of the gain variation decreases asymptotically towards zero. Thus, the gain crosstalk in FSK produces residual amplitude modulation which is less harmful than an equivalent amplitude modulation in an ASK system. To further quantify the effect of this crosstalk on an FSK system, the detection scheme has to be taken into consideration (for example, double...
filter envelope detection or limiter-discriminator detection) which is beyond the scope of this paper. It suffices to point out that the induced amplitude crossstalk in an FSK system is less harmful than an ASK system and in contrast it can be reduced to a negligible value by increasing channel separation. However, when considering angle modulation, we have to study the effect that the carrier density modulation has on the real part of the refractive index, because this will lead to phase crossstalk which may be very important for phase shift keying. Channel crossstalk of $-30 \,$ dB has been observed experimentally when the amplifier is used with constant-envelope modulation [11].

IV. CONCLUSIONS

It has been previously established that, in the small-signal case, a traveling wave optical amplifier has slightly different channel gains when two channels are amplified simultaneously. We have extended this by incorporating the effects of gain saturation, and we have found that when practical output powers are obtained from the amplifier, the gains can differ by as much as 10 dB. We have studied the dependence of channel gains on the important operating parameters such as channel spacing and input power levels. We have also found that if the 1-dB gain compression criterion is used in an ASK system, it would be necessary to operate the amplifier with output power 5 dB below the saturation level for a channel separation $\approx 2$ GHz.

REFERENCES


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